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Highlights

- Introduction of a marketing-operations interface model of new product updates.
- Model incorporates core elements of two seminal diffusion and lot-sizing models.
- Optimal introduction timing of product generations, price and production schedule.
- Performance measures include profitability and pace of new product introductions.
- Impact of changes in model parameters on performance measures is examined.
A dynamic marketing-operations interface model of new product updates

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Abstract

Two critical success factors for nearly any firm are the introduction of new products and the integration of the marketing and operations functions within the business enterprise. The series of products being considered are associated with sequential non-disruptive innovations in technology, but disruptive in fashion. The study presents a model that integrates and builds upon the popular dynamic Bass model for new product diffusion in marketing and the Wagner and Whitin dynamic lot-sizing model, a seminal model in operations management. The end result is a model that simultaneously determines the optimal timing for the introduction of new product generations, pricing, production timing and produced quantities. This model is then used to examine the impact of variations in marketing and operations parameters on both optimal profit and optimal product lifecycle length. The study finds that larger profit and a faster pace of new product introductions are generally associated with faster diffusion, lower price elasticity, larger market potential, lower new product introduction cost and more costly consumer products.

Keywords: Marketing; Diffusion of innovations; Lot sizing; New-product introduction timing; Sensitivity analysis

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1. Introduction

The product life cycles of high technology electronic systems including mobile phones, personal computers are becoming shorter due to continuous innovation in design and technology. This triggers companies to introduce multiple-generation product lines (MGPL) where the original product enters the market first after which successor products are introduced over time. In MGPL, each new generation offers new features, improved usability and appearances but the core functionality of the original product remains unchanged in successor generations (Kilicay-Ergan et al., 2015).

Verganti (2009) asserts that needs of people are not only satisfied by technological functionality, but also in the form of meanings that could be designed. When successful in innovating the meaning of products, the longevity of the product is much higher than when just innovating the function. In this way, the company can escape the innovation race for a longer period of time. The examples mentioned below demonstrate the above assertion more clearly.

Before the advent of the electronic watch in the 1970s, watches were considered jewelry; they were mainly sold in jewelry stores and were primarily made in Switzerland. When digital technology emerged, early applications tried to substitute the mechanical movements with the new components, without changing the meaning. These watches were primarily made in Japan. The Japanese dominated the watch industry until the Swatch Company revitalized the Swiss watchmaking industry through a radical meaning change: watches as fashion. Swatch was marketed as a fashion accessory. Whereas people used to own only a single watch, Swatch encouraged them to own multiple watches, just as they owned multiple shoes, belts, ties, and scarves. They encouraged
their customers to change their watches to match their clothes. Within ten years, the Swatch Group became the world’s leading manufacturer of watches (Norman and Verganti, 2014).

Technological innovations abound in electronic products, but in the boutique electronics industry innovations in fashion are also very important. High levels of demand for a product may be due to some technological innovation such as enhanced functionality of the product, but increased demand is often attributed to fashion. Most boutique electronics are based on technology that was state-of-the-art not very long ago but has since become relatively easy to replicate. If one ignores fashion, as the technology becomes more commonplace the product’s margin would decrease. However a firm may be able to maintain high margins by leveraging fashion appeal. A prime example of this is the Motorola RAZR. When the Motorola RAZR was first introduced, it sold for around $500 despite the fact that it had functionality similar to other phones which sold for much less (Cuneo, 2006).

This scenario is not limited to mobile phones. The MP3 format has been around since the mid-nineties and portable MP3 players have been readily available since 1999. Apple introduced the iPod in late 2001. Over the ensuing six years iPod sales totaled over 100 million units with sales of over 50 million units in 2007 alone (Apple Computer, Inc., 2007). From its inception in 2001, around 20 different models have been developed not counting multiple colors of otherwise identical iPods. The 20 models fall into essentially 5 types of iPods, introduced in the market at regular time intervals. This product diversity is due in large part to fashion.
According to Tang (2010) “The goal of marketing is to create demand, and the objective of operations management is to match and fulfill demand. Thus, for every firm, these two functions are intimately connected.” To further this end, this study presents a model that incorporates two seminal works in the fields of marketing and operations: the Bass model and the Wagner and Whitin lot-sizing method. The article that first introduced the Bass model (1969), was named one of the ten most influential articles in the history of Management Science (Bass, 2004). The article that first introduced the Wagner and Whitin lot-sizing method (Wagner and Whitin, 1958a), has spawned an entire field of research within operations management. One of the branches of the diffusion of innovation literature concerns the timing of the introduction of subsequent generations in a family of products. Norton and Bass (1987; 1992) extend the original Bass model to consider multiple generations within a product family where repeat purchases across generations may occur. Norton and Bass (1987) empirically test their model using data on microchips and find that it has a good fit. Norton and Bass (1992) extend the model to include repeatedly purchased products such as pharmaceuticals and disposable diapers. The authors conclude that the coefficients of innovation and imitation for a given product do not change significantly from generation to generation. Mahajan and Muller (1996) and Kim et al. (2000) arrive at similar conclusions whereas counter conclusions were provided by Pae and Lehman (2003). These counter conclusions were subject to criticism by Van den Bulte (2004).

From an operations management standpoint, the current study is an extension of Wagner and Whitin’s (1958b) classic dynamic lot-sizing model. Wagner and Whitin assume that demand in each period is known and fixed a priori. This leads to an optimal
production schedule that defines the timing of production. The Wagner and Whitin’s (1958b) model focuses on cost minimization. Implicit in this is the assumption that price is fixed and thus can be ignored. In other words, profit can be maximized by minimizing cost because revenue is a fixed constant. The model presented in this study differs from Wagner and Whitin’s (1958b) model in important ways. First, similar to the work of Thomas (1970) which in turn depends, in its solution, on theorems first presented by Wagner and Whitin (1958a), the current model relaxes the assumption that price is a fixed constant. The current model allows for the fact that changing prices can have an impact on demand and thus profit. In doing so, the structure of the formulated model changes from cost minimization to profit maximization. Second, the current model incorporates new product introduction timing decisions.

Cross-functional interface encompasses the organizational structures, tactics and policies adopted by firms to manage the information flow, the conflicts and the mutual objectives between two distinct functional areas (Moenaert and Souder, 1996). Marketing and operations as functional areas represent the key value adding areas of the modern business enterprise. It is these areas that are influential in specifying what is produced, how it is produced and actually delivering goods and services to customers. However, operations managers are often evaluated on cost performance while marketing managers are often rewarded based on revenues. Because of the difference in their reward systems, among other reasons, the conflict between these two important functional areas have been scrutinized in the literature (e.g., Piercy, 2007; Shapiro, 1977) with how to reduce the conflict also being addressed (e.g., Artz et al., 2012; Omurgonulsen and Surucu, 2008; Piercy, 2010). This research assumes that the above two functions are fully
integrated within the business enterprise to optimize performance (Juttner et al., 2007; Hilletofth, 2011).

The way for phasing out an older product generation and introducing a new one is referred to as “product rollover”. There are two main strategies associated with this process. According to the “solo-product roll strategy”, a new generation of the product fully replaces the older generation. Regarding the “dual-product roll strategy”, on the other hand, old and new generations coexist in the market until sales of the old generation(s) drop drastically. This paper adopts the complete replacement “solo-product roll strategy”. Liao and Seifert (2015) provide a variety of examples from industry in support of the solo-product roll strategy. As in Carrillo (2005 a, b) demand for each generation is assumed governed by a modified version of the Bass (1969) diffusion model.

The present study provides sensitivity analysis results related to a decision model that incorporates, for the first time in the literature, product introduction timing, pricing and production scheduling in a multi-generational product scenario. Sensitivity analysis is the study of how the uncertainty in the output of a mathematical model or system (numerical or otherwise) can be apportioned to different sources in its input parameters (Saltelli, 2002; Saltelli et al., 2008). Wagner (1995) mentions the following main compelling motives for conducting sensitivity analysis: (i) understanding the sensitivity of a model’s outputs to simultaneous variations in several parameters, (ii) assessing whether variations in particular parameters do or do not have significant impact on the optimal value of the objective function, (iii) ascertaining the relative influence of model parameters, and (iv) bringing insights and results needed by managers.
This study considers the situation of a monopolistic manufacturer planning to introduce \( G \) successive generations of a product. The manufacturer aims to determine the life cycle of each generation together with the related production schedule, production quantities, and selling prices to meet a deterministic price-dependent demand so as to maximize profit per unit time over a sufficiently long planning horizon. The first generation is assumed to be ready for introduction at time zero at which time, the initial inventory is also zero. After the first generation is introduced, product development efforts for the second generation, scheduled to be introduced in the market at time \( T + 1 \), begin. By time \( T \), all units of the first generation are sold and its ending inventory at time \( T \) is zero. This sequence of events repeats itself until the last generation is introduced at time \((G-1)*T + 1\) and all of the last generation’s units are sold by time \( GT \) so that ending inventory is again zero. Demand for each generation is assumed to be governed by a modified version of the Bass (1969) model in which price affects the diffusion rate in a multiplicative fashion.

The research questions associated with the above interdisciplinary problem that has not been considered before in the literature in such breadth are:

1. How to mathematically formulate the above problem and arrive at optimal solutions for the life cycle length together with the related optimal profit per unit time.

2. What are the marketing parameters the variations of which have significant/insignificant impacts on optimal product life cycle length and optimal profit per unit of time?
(3) What are the operations parameters the variations of which would have significant /insignificant impacts on optimal product life cycle length and optimal profit per unit of time?

(4) What is the relative influence of model parameters on the above two equilibrium quantities?

The research is thought to be applicable to products in an industry that experiences minor technological innovations and which product success is due in part to fashionability. Product life cycles are relatively short and not limited to coincide with technological breakthroughs. Instead, it is assumed that new products are introduced in a regular pattern. High-end cell phones and other boutique electronics items, such as personal organizers or iPods, are examples of these types of products.

The rest of the paper is organized as follows: Section 2 provides a related background. In section 3, the demand function is developed and the mathematical program for the problem is briefly introduced. In Section 4, an experimental design to examine the sensitivity of profit and lifecycle length to changes in model parameters is highlighted. The related results are shown in Section 5. Finally, Section 6 summarizes the paper, illustrates its contribution, managerial implications and directions for future research. To improve exposition, an additional literature review is included in Appendix A. Analytical solution details are provided in Appendix B, whereas related computer solution methodology details are included in Appendix C. The three appendices are included in a Supplementary Material component.

2. Background

Given the problem statement highlighted in the introduction section, a literature
review relevant to the scope and purposes of the present study is introduced. Special attention is given to reviewing models integrating marketing and operations management. To highlight the contribution of the study, the article is positioned afterwards with respect to the reviewed literature.

2.1. Literature review

The present study is based on a model that considers lot-sizing, diffusion of innovation and product introduction timing within the context of new multi-generation products context. Related relevant literature is reviewed below.

2.1.1. Lot-sizing models incorporating successive generations

Though there are some models that use the diffusion process of a single generation product to formulate the optimal inventory policy (Kurawarwala and Matsuo, 1996; Chern et al., 2001), there is a scarcity of models that study the effect of innovation and substitution on the optimal policy. Li et al. (2013) consider the demand for multiple successive generations of products and develop a population-growth model that allows demand transitions across multiple product generations. The focus is on developing an inventory policy for a new product under the condition of diminishing demand when its substitutive product enters into the market. Ke et al. (2013) propose an integrated inventory (supply) and diffusion (demand) framework and analyze how inventory costs influence the introduction timing of product-line extensions, considering substitution effect among successive generations. The authors show that under low inventory costs the “now or never rule” is optimal whereas the sequential introduction becomes optimal as the inventory costs become substantial. Chanda and Aggrawal (2014) derive optimal inventory policies for successive generations of a high technology product. The model is
based on the assumption that technological advancements do not essentially imply that existing generation products will be withdrawn from the market immediately. An additional literature review related to single generation products is found in Appendix A.

2.1.2. Diffusion models incorporating successive generations

Norton and Bass (1987, 1992) extend the original Bass (1969) model to consider multiple generations within a product family where repeat purchases across generations may occur. The authors conclude that the coefficients of innovation and imitation for a given product do not change significantly from generation to generation. Wilson and Norton (1989) also consider both diffusion and substitution through an extension of the model presented by Kalish (1985). One of Wilson and Norton’s main findings is that when the planning horizon is sufficiently long, it is optimal for a monopolist to either introduce a second generation as soon as possible or not to introduce it at all. Mahajan and Muller (1996) propose a model that simultaneously captures the adoption and substitution patterns for successive generations of a durable technological innovation. The authors find that it is optimal to either introduce a new product as soon as possible or to delay introduction until the growth phase of the current generation has ended. Kim et al. (2000) propose a general model framework that incorporates both inter-category dynamics and intergenerational substitution effects for a multiproduct market. The authors estimate their system of equations using data from the Korean and Hong Kong wireless communication market. Stremersch et al. (2010) observe that while the diffusion literature concludes that more recently introduced products show faster diffusion than older ones, the literature argues that diffusion parameters remain constant across generations. Upon controlling for the passage of time, the paradox has been
resolved through showing that intergeneration acceleration occurs in the time to takeoff but not in the diffusion parameters. The models discussed above do not consider marketing mix variables. Both normative (e.g., Bayus, 1992; Padmanbhan and Bass, 1993; Li and Graves, 2012) and empirical (e.g., Speece and MacLachlan, 1992; 1995; Danaher et al., 2001; Orbach and Fruchter, 2011) research have considered price in a product substitution setting. Additional literature review pertaining to the diffusion of single generation products is found in Appendix A.

2.1.3. Introduction timing models incorporating successive generations

The models reviewed in the above two sections assume that the times of entry of successive generations are determined exogenously. While several articles have examined timing decisions in a diffusion only scenario without pricing decisions (Carrillo, 2005a,b; Krankel et al., 2006; Druehl et al., 2009; Koca et al., 2010; Lia and Seifert, 2015), there is a scarcity of models that consider the joint effect of timing and pricing decisions. Gaimon and Morton (2005) model changeover flexibility decisions in the context of a firm’s market entry strategy for successive product generations of short life cycles produced in a single facility. The authors also derive the optimal pricing policy for each product generation as a function of the firm’s market entry strategy and manufacturing efficiency. Lim and Tang (2006) develop an analytical model to analyze the profits associated with two product rollover strategies: single-product rollover and dual-product rollover. The authors determine for each strategy the prices of both products as well as the time to launch the new product and the time to phase out the old product. Seref et al. (2016) develop an analytical model of coordinated product timing and pricing decisions when there are two generations of a new product under consideration. Factors
used to derive the timing and pricing decision include the unit sales and cost relationships for each generation as well as new product development costs for introducing the next generation of products. An additional literature review relative to single generation products and other pertinent issues are found in Appendix A.

2.2. Positioning and contribution of study

The literature reviewed above and/or in Appendix A reveals few lot-sizing methodologies that are appropriate for innovative products such as Hausman and Peterson (1972), Hartung (1973), Bitran et al. (1986), and Matsuo (1990). All of these articles differ from the current study in that they present heuristics rather than exact solutions. Hausman and Peterson (1972) present three solution heuristics. These heuristics treat each product similarly to the classic newsvendor problem then use Lagrange multipliers to allocate capacity. Hartung (1973) presents a heuristic that is appropriate when demand is stochastic. Bitran et al. (1986) and Matsuo (1990) both use two-phase heuristics to solve the problem. In the first phase, an aggregate plan is produced where aggregation is at the product family level. In the second phase, the plan is disaggregated to individual products. The model presented in Appendix B is compared to that of Thomas (1970). It extends the work of Thomas in several important ways. Chief among these extensions are the use of a well-defined demand function (Bass, 1969), the use of a realistic price response function (Bass, 1980), and the incorporation of new product introduction timing into the model.

The reviewed diffusion of innovation literature falls into two categories: those articles that consider multiple generations, and those articles that consider the effect of price on demand. The current study sits at the intersection of these two bodies of literature in that it simultaneously considers both multiple generations and the
relationship between price and demand. In addition to these considerations, the formulated profit maximization model also considers the production and inventory costs together with product development costs. Furthermore, the current study investigates the impact of various diffusion and cost parameters on the optimal introduction timing of successive generations and the related optimal profit.

The reviewed literature on introduction timing indicates that Carrillo (2005a,b) are two of the most similar to the current study. Both models assume that generations of products have equal planned product life cycle lengths, that only a single generation is present in the market at any one time, and that product development and introduction costs for different generations are equal. Our study differs in that it does consider optimal pricing or production scheduling decisions. A second similar article is Kurawarwala and Matsuo (1996). Like the model presented in Appendix B, it incorporates many inventory management concepts with a forecasting model based on the Bass model. Major differences between the two models include the absence of price from the model presented by Kurawarwala and Matsuo (1996) as well as their assumptions that procurement lead times are greater than product lifecycles. A third article that is akin to the current study is Krankel et al. (2006). Both models seek to optimize new product introduction timing in the presence of incremental technological innovations. One major difference is that Krankel et al. (2006) explicitly model these innovations using a stochastic process whereas the current model captures innovation implicitly. Another major difference is that the current study also simultaneously optimizes production scheduling and pricing while these issues are not considered by Krankel et al. (2006).
In sum, although there is significant research in lot-sizing, diffusion of innovations, and new product introduction timing, to the best knowledge of the authors, the model presented in this article (Section 3 and Appendix B) is the first to consider optimum product introduction timing, pricing, and production scheduling decisions in a multi-generational product scenario, and therefore attempts to fill a gap in the literature.

3. Model description

Four groups of decision variables are associated with the problem stated earlier. The first is the price to charge in each period. The second concerns when to produce. The third concerns the quantity to produce when production occurs. The fourth decision variable concerns when to introduce the next product generation.

3.1 Model main assumptions

(i) The products represent subsequent generations in the same family of products in an industry that experiences minor technological innovations and in which product success is due in part to fashion (Verganti, 2009).

(ii) The planning horizon is sufficiently long and product life cycles are relatively short that several generations of the product family are planned.

(iii) The producer is following a solo-product roll strategy (Billington et al., 1998). This means that the inventory of one product iteration is exhausted at the same time that the next product iteration is introduced and ready for sale.

(iv) Demand for each product-iteration is governed by a modified version of the Bass (1969) diffusion model that incorporates price.

(v) Various demand and cost characteristics being considered do not change from one product iteration to the next.

(vi) No backlog of demand is maintained and any unmet demand is lost.
(vii) The manufacturer is a monopolist or at least the dominant rival of a market that is made up solely of the dominant rival and smaller competitors are not large enough to affect the market in a meaningful way.

3.2 Model highlights

As stated in the first section, the demand curve used in this model is based on the Bass model. The Bass model is based on two behavioral forces: innovation and imitation. The portion of demand that occurs independently of the cumulative demand is associated with the coefficient of innovation, \( p \). The portion of demand that varies with cumulative demand is associated with the coefficient of imitation, \( q \). When a product is first introduced, only innovators purchase the product. In subsequent periods, some portion of the total demand is caused by additional innovators purchasing the product and some portion is caused by imitators purchasing the product. Over time, the influence of innovation decreases while the influence of imitation increases. The likelihood of purchase at time \( t \) given that no purchase has yet been made is given by

\[
\frac{f(t)}{1 - F(t)} = p + q^* F(t),
\]

where \( f(t) \) is the likelihood of purchase at time \( t \) given by \( dF(t)/dt \). Solving this differential equation when \( F(0) = 0 \) yields Equation (1).

\[
F(t) = \frac{1 - e^{-bt}}{1 + ae^{-bt}}, \tag{1}
\]

where \( a \) is defined as the ratio of \( q \) to \( p \) and \( b \) is defined as the sum of \( p \) and \( q \).

Demand occurring during period \( t \) is found by taking the difference between Equation (1) and itself for the values of \( t \) and \( t-1 \) and multiplying the result by the size of the market. Thus absent of any price impact, which is discussed below, demand in period \( t \) is defined by Equation (2) where \( m \) is the total size of the potential market.
\[ D_\beta(t) = m^* [F(t) - F(t-1)]. \]  

(2)

The subscript \( \beta \) represents the fact that this is basic demand prior to consideration of a price impact.

The Bass model has been modified and applied in a wide range of areas. Several authors have modified the Bass model in various ways to incorporate the influence of price on demand. The literature contains two main approaches for the incorporation of price as a decision variable. The first approach is to model the current demand rate as a function of the current price and the current installed base [Equation (2)]. The second main approach is to model demand as an exogenous life cycle curve which is a function of time multiplied by a price response function. The approach used in this study is consistent with the second approach followed by Bass (1980), Bass and Bultez (1982) and Mesak (1990).

Again, the literature contains two main price response functions. The first function results in a price elasticity of demand that is a linearly increasing function of price (e.g., Robinson and Lakhani, 1975 use the price response \( e^{BP} \), where \( B \) is a constant price sensitivity parameter) while the second function results in a price elasticity of demand that is constant. This second type of response function has been used by Bass (1980), Bass and Bultez (1982), Mesak (1990), and Jain and Rao (1990). A constant elasticity price response function of the form \( \left( \frac{P}{P_0} \right)^{-\eta} \) is used in the current study, where \( \eta \) is a constant price elasticity parameter, and \( P_0 \) is a fixed base price. Bass, Krishnan, and Jain (1994) find empirically that models employing this price response function tend to exhibit better fit. The incorporation of price in model (2) produces model (3), or
\[ D(t, P_t) = D_{\mu}(t) \left( \frac{P_t}{P_0} \right)^{-\eta}. \]  

That is
\[ D(t, P_t) = m^* \left[ P(t) - P(t-1) \right]^* \left( \frac{P_t}{P_0} \right)^{-\eta}. \]

Note, that in order to make notation simpler, \( D(t, P_t) \) will be written as \( D_t \) in the remainder of this study.

The four marketing oriented model parameters presented so far are \( a, b, m, \) and \( \eta \). In addition to these, there are four operations oriented model parameters as well, \( H, S, V, \) and \( I \). These represent holding cost per unit-period, setup cost per production run, variable cost per unit, and new product introduction cost respectively.

The problem is formulated as a Mixed Integer Programming Model. The cost structure, along with the demand pattern discussed earlier, defines the dynamic lot-sizing with price elasticity and new product introduction, or the DLPEND model. The objective of the DLPEND model is to maximize average profit per period. The reason average profit per period is used as an objective instead of total profit is that one of the decision variables is the number of periods in which to offer the product before introducing the next generation and suspending production of the current generation. Therefore, the objective function for the model is given by Expression (4).

\[
Max \left( \sum_{t=1}^T ((P_t - V) \cdot D_t - S \cdot \sigma_t - H \cdot Inv_t) - I \right),
\]

where the decision variable \( T \) is the number of time periods to produce the product, the decision variable \( P_t \) is the price in period \( t \), the binary decision variable \( \sigma_t \) takes on a value of one when production occurs in period \( t \) and a value of zero when no production
occurs, and \( \text{Inv}_t \) is the total inventory at the end of period \( t \), and \( t \) takes on integer values from 1 to \( T \).

Inventory on hand at the end of period \( t \), \( \text{Inv}_t \), is governed by equation (5).

\[
\text{Inv}_t = \text{Inv}_{t-1} + X_t - D_t \quad \text{for} \quad t = 1 \text{ to } T.
\] (5)

Note that \( X_t \) is a decision variable that represents the production quantity in period \( t \). Also note that \( \text{Inv}_0 \) is equal to zero.

A second type of constraint upon expression (4) ensures that the production is sufficient to meet demand. The production constraints are shown in equation (6).

\[
\sum_{t=1}^{i} X_t \geq \sum_{t=1}^{i} D_t \quad \text{for} \quad i = 1 \text{ to } T.
\] (6)

Constraints (6) ensure that the cumulative production up to a given period is sufficient to meet the cumulative demand up to that period.

A third type of constraint upon expression (4) ensures that setups occur in each period where the production quantity is positive. The setup constraints are shown in (7).

\[
m \ast \sigma X_t \geq 0 \quad \text{for} \quad t = 1 \text{ to } T.
\] (7)

These constraints ensure that setups occur in every period in which production occurs.

As stated previously, the solution to this optimization problem is an extension of the Wagner and Whitin (1958a) lot-sizing method. The Wagner and Whitin method finds the optimal production schedule in terms of timing and quantities given a fixed demand schedule. Demand in this schedule can vary from period to period in any way, but within a given period must be a fixed constant. Thomas (1970) presented an extension of the Wagner and Whitin method that relaxed this last assumption. His new method allows for a demand in a given period to vary depending on price in that period. The method is applicable so long as \( D_t \) is a deterministic function which, as can be seen in Equation (3),
is the case in this study.

As mentioned earlier, Thomas (1970) allows demand to vary deterministically in a period based on price. Suppose in period one no inventory is held. Then the gross profit function is given by Expression (8).

\[(P_1 - V)^* D_t.\]  

(8)

The optimal value of \(P_1\) is found by equating to zero the first derivative of Expression (8) with respect to price. Recalling that \(D_t\) is ultimately a function described by Equations (1) and (2), the optimal value of \(P_1\) is given by Equation (9).

\[P_1 = \frac{V^* \eta}{\eta - 1}.\]  

(9)

Similar to Bass (1980), \(P_0\) is equal to price in the first period. Therefore Equation (9) also defines \(P_0\). Note that \(\eta\) is constrained in this study to be strictly greater than one.

One thing to note is that Equation (9) is only valid in periods in which no inventory is held. The presence of inventory holding costs leads to a slightly different gross profit function and therefore a slightly different optimal value of \(P_t\). The optimal solution to \(P_t\) in the general case is given by Equation (10).

\[P_t = \frac{(V + HN_t)^* \eta}{\eta - 1},\]  

(10)

where products are held in inventory for \(N_t\) periods before being sold in period \(t\).

With the optimal values for the various \(P_t\) known, Equation (2) can be used to calculate the associated demands. Then Thomas’s (1970) method can solve for the optimal production schedule. More details about the analytical solution methodology are found in Appendix B. Details of the related computer solution methodology are found in
Appendix C.

4. Numerical experiments

The purpose of this section is to design numerical experiments that aim at analyzing the sensitivity of both optimal profit per unit time and optimal product lifecycle length (reciprocal of pace, or clock speed as often mentioned in the literature) to changes in model parameters.

As stated in the previous section, the model consists of eight parameters, four that are essentially marketing centric parameters \((a, b, m \text{ and } \eta)\) and four that are essentially operations centric parameters \((H, S, V \text{ and } I)\). The first parameter to be considered is \(a\), which is the ratio of the coefficient of imitation to the coefficient of innovation, or \(q/p\). Bass (1969) finds that \(a\) ranges from 9.0 to 82.4. In a study related to the diffusion of short life-cycle products, Kurawarwala and Matsuo (1996) reports values of \(a\) as small as 1.7. The products being considered in the current study have relatively short lifecycles; therefore, the range of values for \(a\) considered in the current study is 2 to 20.

The second parameter to be considered is \(b\), which is the sum of the coefficients of innovation and imitation, or \(p + q\). Bass (1969) finds that \(b\) ranges from 0.19 to 0.68. As seen in Appendix A the expression for the time of peak demand, \((1/b) \ln a, b\) is negatively related to product lifecycle length (Bass, 1969). Note that Bass (1969) uses annual data while the current study uses weekly data. In order to derive a \(b\) value appropriate for weekly data, a \(b\) value based on annual data must be divided by the number of weeks in a year (Putsis, 1996; Non et al., 2003). This would result in a range of values from 0.004 to 0.013. In their study of short lifecycle products, Kurawarwala and Matsuo (1996) report values of \(b\) as large as 0.4676 when using
monthly data. This is equivalent to a value of 0.12 after converting it to a weekly value. This value is significantly higher than any other values that were found in a search of the literature. Therefore the range of values for \( b \) considered in the current study is 0.01 to 0.1 and is associated with weekly data.

It is worthy to mention at this junction that we consider the ratio \( q/p \) to be one parameter and the sum \( p + q \) to be another (also used by Krishnan et al., 1999; Druehl et al., 2009; Koca et al., 2010), due to the way the ratio and sum define diffusion as seen by Equation (1). The ranges assigned to the two parameters were not only inspired by the empirical results reported in Bass (1969), Kurawarwala and Matsuo (1996) and Pae and Lehmann (2003), but also by the ranges of the same parameters employed in the numerical analyses of Krishnan et al. (1999), Druehl et al. (2009) and Koca et al. (2010).

The third parameter to be considered is \( m \), which is the total population of potential customers. In this regard, the upper bound on \( m \) is set near 10% of the population of the United States. Such a percentage appears plausible in practice. For example, when Sony introduced the PlayStation 2, the highly popular original PlayStation that was launched only four years earlier had an installed base of 30 million in the United States (Peterson, 2004). The lower bound on \( m \) is set between 1.5% and 2% of the population of the United States (314,000,000). Therefore the range of values for \( m \) considered in the current study is 6,000,000 to 30,000,000.

The fourth parameter to be considered is \( \eta \), which is the price elasticity parameter. Bass (1980) finds values of \( \eta \) as large as 8.02. Therefore, the range of values for \( \eta \) considered in the current study is 2 to 8.

The fifth parameter to be considered is \( H \), which is holding costs measured in
dollars per unit per week. The operations management literature does not normally
discuss holding costs in dollars, but in terms of a percentage of the product’s selling
price. Many sources in the production management literature suggest that holding costs
are between 15% and 40% of price per unit per year (Rubin et al., 1983; Jordan, 1989;
Raman & Kim, 2002). The products being considered in the current study are expected
to sell for around $100 to $200. Therefore, the range of values for $H$ considered in the
current study is $0.10$ to $1$ per unit per week.

The sixth parameter to be considered is $S$, which is setup cost. The production
management literature does not normally discuss setup costs in dollars because it is
highly context specific. The literature suggests two ways of addressing this difficulty.
Some authors, such as Berry (1972), set $S$ indirectly by considering the economic time
between orders (TBO). One of the most commonly used values for economic TBO is two
weeks (Benton & Whybark, 1982; Lin, Krajewski, Leong, & Benton, 1994). Economic
TBO, measured in years, is equal to the ratio of the EOQ to annual demand. When using
the values of $H$ mentioned in the previous paragraph, an expected annual demand of
around one million units, and a target TBO of around two weeks, this method suggests a
range of values for $S$ of around $5,000$ to $30,000$. A simpler approach that does not rely
on demand is proposed by Wemmerlov (1982, p. 469). Wemmerlov suggests that the
ratio of $S$ to $H$ be used and employs a range of values from 25 to 600. Note that
Wemmerlov’s $H$ is the annual holding cost whereas the $H$ used in the current study is
weekly holding cost. When using the values of $H$ mentioned in the previous paragraph,
Wemmerlov’s range of ratios suggest a range of values for $S$ of $250$ to $30,000$.
Therefore, the range of values for $S$ considered in the current study is $3,000$ to $30,000$
per unit per week.

The seventh parameter to be considered is $V$, which is variable cost per unit. According to Fisher (1997) innovative products have gross profit margins (GPM) between 20% and 60%. Based on this and selling prices in the range from $100 to $200, the range of values for $V$ considered in the current study is from $50 to $100 per unit.

The eighth and final parameter to be considered is $I$, which includes research and development as well as other new product introduction costs. Ulrich and Eppinger (2004) suggest that research and development costs for new products tend to be less than 5% of total revenue. As stated previously expected annual demand is around one million units and prices are expected to be $80 to $150 per unit. Also product lifecycles are expected to be around three years. Therefore, the range of values for $I$ considered in the current study is $4,000,000 to $25,000,000. The factor ranges discussed above are divided such that each of the eight parameters is set at four factor levels in this study in order to keep the computational effort at a manageable level. These factor levels are shown below in Table 1 where capital M represents millions and capital K represents thousands.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Factor Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>2, 8, 14, 20</td>
</tr>
<tr>
<td>$b$</td>
<td>0.01, 0.04, 0.07, 0.10</td>
</tr>
<tr>
<td>$m$</td>
<td>6M, 14M, 22M, 30M</td>
</tr>
<tr>
<td>$\eta$</td>
<td>2, 4, 6, 8</td>
</tr>
<tr>
<td>$H$</td>
<td>0.1, 0.4, 0.7, 1.0</td>
</tr>
<tr>
<td>$S$</td>
<td>3K, 12K, 21K, 30K</td>
</tr>
<tr>
<td>$V$</td>
<td>90, 110, 130, 150</td>
</tr>
<tr>
<td>$I$</td>
<td>4M, 11M, 18M, 25M</td>
</tr>
</tbody>
</table>

The full factorial design related to the parameters depicted in Table 1 includes $4^8$, or 65,536, combinations (treatments). For each combination, it is relatively
straightforward to use the method described earlier to find the optimal values of the maximum profit per unit time and the optimal product lifecycle length. Two multiple linear regressions are performed after all treatments are completed with the eight parameters listed in Table 1 as independent variables in both regressions. Maximum profit per period and optimal product lifecycle length are the two dependent variables. Prior to performing these regression analyses, all variables are centered around their respective means in order to avoid potential multicollinearity problems. The magnitude of the influence of a given independent variable on the dependent variable is measured by the absolute value of the t-statistic related to the estimated coefficient of the independent variable within a multiple linear regression framework. This approach is consistent with the global sensitivity analysis method proposed by Wagner (1995).

5. Numerical results

Table 2 summarizes the results when profit per period is the dependent variable. The $R^2$ value associated with this model is 0.389.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>-47.320**</td>
</tr>
<tr>
<td>$b$</td>
<td>109.910**</td>
</tr>
<tr>
<td>$m$</td>
<td>91.873**</td>
</tr>
<tr>
<td>$\eta$</td>
<td>-133.549**</td>
</tr>
<tr>
<td>$H$</td>
<td>-0.053</td>
</tr>
<tr>
<td>$S$</td>
<td>-0.138</td>
</tr>
<tr>
<td>$V$</td>
<td>34.479**</td>
</tr>
<tr>
<td>$l$</td>
<td>-2.595*</td>
</tr>
</tbody>
</table>

***: $p < 0.001$; **: $0.001 \leq p \leq 0.1$, otherwise $p > 0.1$

Table 3 summarizes the results when product lifecycle length is the dependent
variable. The $R^2$ value associated with this model is 0.372.

Table 3: Multiple Linear Regression Results for Product Lifecycle Length

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>74.262**</td>
</tr>
<tr>
<td>$b$</td>
<td>-182.286**</td>
</tr>
<tr>
<td>$m$</td>
<td>-5.341**</td>
</tr>
<tr>
<td>$\eta$</td>
<td>5.215**</td>
</tr>
<tr>
<td>$H$</td>
<td>0.015</td>
</tr>
<tr>
<td>$S$</td>
<td>0.032</td>
</tr>
<tr>
<td>$V$</td>
<td>-1.686*</td>
</tr>
<tr>
<td>$I$</td>
<td>5.008**</td>
</tr>
</tbody>
</table>

‘**’: $p < 0.001$; ‘*’: $0.001 \leq p \leq 0.1$, otherwise $p > 0.1$

All values in Table 2 and Table 3 relate to profit per period and product lifecycle length in the direction predicted by theory, where such theory exists. Profit behaves as expected in relation to $m$, $\eta$, $H$, $S$, and $I$ although $H$ and $S$ are insignificant. Profit is negatively related to $a$ and positively related to $b$. This is as expected because the dependent variable here is profit per period and product lifecycle length is positively related to $a$ and negatively related to $b$ (Bass, 1969). Finally, profit is positively related to $V$. Although this appears counterintuitive, this is also as expected. As can be seen in Equation (8), the optimal price is a positive function of $V$ and as this cost increases price increases by an even greater amount as $\eta$ is greater than one. Furthermore, as in Carrillo (2005), the coefficient of $I$ is negative.

The present study considers also the effect of the two-way interactions among the eight parameters alluded to earlier. Tables 4 and 5 indicate that all the six interaction terms among the four marketing parameters are significant whereas the six interaction terms among the four operations parameters are not. In Table 4, only five out of the 16
interaction terms among the marketing and the operations parameters are significant.

Four out of these five significant interaction terms involve the operations parameter $V$. In Table 5, on the other hand, only two out of the 16 interaction terms among the marketing and operations parameters turned out to be significant.

It is worthy to mention that the interaction terms involving parameter $V$ that are significant with other marketing parameters have the same sign as their main effect in Table 4. This means that an error in estimating any such marketing parameters will have more serious consequences on the assessment of profitability for larger values of parameter $V$ than on the assessment of the same, but for smaller values of the operations parameter.

In short, considering the absolute values of the $t$-statistics, Tables 2 through 5 taken together imply that while the insights gained from the main effects remain robust, such main effects are not sufficient to fully explain the variation in the respective dependent variables. Upon considering the interaction terms, the $R^2$ related to Table 2 has increased from 0.389 to 0.566 whereas the $R^2$ related to Table 3 has increased from 0.372 to 0.410. Furthermore, Tables 4 and 5 demonstrate that the absolute value of the $t$-statistic of any interaction term is much smaller than the absolute value of the same pertaining to either of its separate first order (main effect) components.

Table 4: Multiple Linear Regression with Interactions for Profit

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>-55.761**</td>
</tr>
<tr>
<td>$b$</td>
<td>129.515**</td>
</tr>
<tr>
<td>$m$</td>
<td>108.261**</td>
</tr>
<tr>
<td>$\eta$</td>
<td>-157.371**</td>
</tr>
<tr>
<td>$H$</td>
<td>-0.062</td>
</tr>
<tr>
<td>Parameter</td>
<td>t</td>
</tr>
<tr>
<td>-----------</td>
<td>----</td>
</tr>
<tr>
<td>$S$</td>
<td>-0.163</td>
</tr>
<tr>
<td>$V$</td>
<td>40.630**</td>
</tr>
<tr>
<td>$l$</td>
<td>-3.058*</td>
</tr>
<tr>
<td>$a \times b$</td>
<td>-33.989**</td>
</tr>
<tr>
<td>$a \times m$</td>
<td>-28.921**</td>
</tr>
<tr>
<td>$a \times \eta$</td>
<td>42.133**</td>
</tr>
<tr>
<td>$a \times H$</td>
<td>0.038</td>
</tr>
<tr>
<td>$a \times S$</td>
<td>0.005</td>
</tr>
<tr>
<td>$a \times V$</td>
<td>-10.873**</td>
</tr>
<tr>
<td>$a \times l$</td>
<td>0.150</td>
</tr>
<tr>
<td>$b \times m$</td>
<td>66.024**</td>
</tr>
<tr>
<td>$b \times \eta$</td>
<td>-95.969**</td>
</tr>
<tr>
<td>$b \times H$</td>
<td>-0.015</td>
</tr>
<tr>
<td>$b \times S$</td>
<td>-0.009</td>
</tr>
<tr>
<td>$b \times V$</td>
<td>24.774**</td>
</tr>
<tr>
<td>$b \times l$</td>
<td>-1.856*</td>
</tr>
<tr>
<td>$m \times \eta$</td>
<td>-78.379**</td>
</tr>
<tr>
<td>$m \times H$</td>
<td>-0.014</td>
</tr>
<tr>
<td>$m \times S$</td>
<td>-0.013</td>
</tr>
<tr>
<td>$m \times V$</td>
<td>20.241**</td>
</tr>
<tr>
<td>$m \times l$</td>
<td>-0.154</td>
</tr>
<tr>
<td>$\eta \times H$</td>
<td>0.001</td>
</tr>
<tr>
<td>$\eta \times S$</td>
<td>-0.002</td>
</tr>
<tr>
<td>$\eta \times V$</td>
<td>-29.389**</td>
</tr>
<tr>
<td>$\eta \times l$</td>
<td>0.165</td>
</tr>
<tr>
<td>$H \times S$</td>
<td>-0.011</td>
</tr>
<tr>
<td>$H \times V$</td>
<td>-0.003</td>
</tr>
<tr>
<td>$H \times l$</td>
<td>0.008</td>
</tr>
<tr>
<td>$S \times V$</td>
<td>-0.002</td>
</tr>
<tr>
<td>$S \times l$</td>
<td>0.005</td>
</tr>
<tr>
<td>$V \times l$</td>
<td>-0.049</td>
</tr>
</tbody>
</table>

‘***’: $p < 0.001$; ‘**’: $0.001 \leq p \leq 0.1$, otherwise $p > 0.1$

Table 5: Multiple Linear Regression with Interactions for Product Lifecycle Length
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>76.617**</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>-188.066**</td>
<td></td>
</tr>
<tr>
<td>m</td>
<td>-5.510**</td>
<td></td>
</tr>
<tr>
<td>η</td>
<td>5.380**</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>0.015</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>0.033</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>-1.739*</td>
<td></td>
</tr>
<tr>
<td>l</td>
<td>5.166**</td>
<td></td>
</tr>
<tr>
<td>ax b</td>
<td>-64.431**</td>
<td></td>
</tr>
<tr>
<td>ax m</td>
<td>1.938*</td>
<td></td>
</tr>
<tr>
<td>ax η</td>
<td>-2.090*</td>
<td></td>
</tr>
<tr>
<td>ax H</td>
<td>-0.020</td>
<td></td>
</tr>
<tr>
<td>ax S</td>
<td>-0.019</td>
<td></td>
</tr>
<tr>
<td>ax V</td>
<td>0.609</td>
<td></td>
</tr>
<tr>
<td>ax l</td>
<td>-1.959*</td>
<td></td>
</tr>
<tr>
<td>bx m</td>
<td>4.650**</td>
<td></td>
</tr>
<tr>
<td>bx η</td>
<td>-4.532**</td>
<td></td>
</tr>
<tr>
<td>bx H</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>bx S</td>
<td>-0.047</td>
<td></td>
</tr>
<tr>
<td>bx V</td>
<td>0.145</td>
<td></td>
</tr>
<tr>
<td>bx l</td>
<td>-4.367**</td>
<td></td>
</tr>
<tr>
<td>mx η</td>
<td>-2.669*</td>
<td></td>
</tr>
<tr>
<td>mx H</td>
<td>-0.008</td>
<td></td>
</tr>
<tr>
<td>mx S</td>
<td>-0.050</td>
<td></td>
</tr>
<tr>
<td>mx V</td>
<td>0.868</td>
<td></td>
</tr>
<tr>
<td>mx l</td>
<td>-2.559</td>
<td></td>
</tr>
<tr>
<td>η x H</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>η x S</td>
<td>0.022</td>
<td></td>
</tr>
<tr>
<td>η x V</td>
<td>-0.848</td>
<td></td>
</tr>
<tr>
<td>η x l</td>
<td>2.480</td>
<td></td>
</tr>
<tr>
<td>H x S</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>H x V</td>
<td>-0.007</td>
<td></td>
</tr>
<tr>
<td>H x l</td>
<td>-0.001</td>
<td></td>
</tr>
<tr>
<td>S x V</td>
<td>-0.011</td>
<td></td>
</tr>
<tr>
<td>S x l</td>
<td>0.010</td>
<td></td>
</tr>
</tbody>
</table>
**V x I** -0.805

‘***’: \( p < 0.001 \); ‘*’: \( 0.001 \leq p \leq 0.1 \), otherwise \( p > 0.1 \)

Given by the absolute value of their t-statistics and in recognition that we have optimized profit through choosing pace (reciprocal of life cycle length), parameters are ordered as \( \eta > p + q \) (or b) > \( m > q/p \) (or a) > \( V > I > S > H \) with respect to their influence on the average profit per period as seen in Table 2.

**6. Summary and conclusions**

This study integrates the fields of marketing and operations by developing for the first time in the literature a model that combines many elements of two classic models from these fields. It extends the marketing literature by extending the Bass (1969) to include operations concerns such as setup and holding costs and lot-sizing concerns. It extends the operations model of Wagner and Whitin’s (1958b) dynamic lot-sizing model by incorporating an empirically justifiable demand function and new product introduction into the formulated problem. The findings of related sensitivity analyses reveal that larger profits (and faster pace of new product introduction) are generally associated with lower price elasticity parameter, faster diffusion speed, larger market potential, more costly consumer products and lower new product introduction cost.

Examination of Tables 2 through 5 in the previous section provides interesting insights. Two of the four operations oriented factors have an extremely small impact on maximum profit. This has significant practical implications for a decision maker facing uncertainty across all eight parameters. Given a moderate range of likely values for each of the parameters, the decision maker should not concern themselves greatly with obtaining better information concerning holding costs, or setup costs. Much more important is obtaining good information on the marketing oriented parameter values and
variable costs as well as new product introduction cost. Our conclusions are, therefore, in conformity with those of Mesak et al. (2015) who consider the interface of marketing represented by advertising and operations represented by an EOQ model in a static setting related to mature markets.

Appendix B presents an optimal solution methodology for the problem formulated in this article. The method described in Appendix B is used to solve a problem facing a particular manufacturing organization through employing the computer solution methodology depicted in Appendix C. Not only would such a model be a valuable decision support tool for making pricing, lot-sizing, and new product introduction timing decisions, it would also provide very valuable information in the form of a sensitivity analysis similar to that found in Section 5. In essence, it would let the organization know which parameters must be accurately assessed and which parameters are keys to higher profits.

To operationalize the model, the user is required to provide a set of eight input parameters related to Table 1. This could be a challenging task in practice as marketing-operations interface models that are used for operations planning in the presence of innovation diffusion dynamics require estimates before any data are observed. To arrive at such estimates, one may adhere to the methods proposed by Little (1970), and Lenk and Rao (1990) enlightened by the literature reviews on diffusion of innovations (Roberts and Latin, 2000) and empirical research in operations management (Gupta et al., 2006).

We acknowledge the limitations posed by the assumptions of our model. The present study assumes that demand is deterministic, variable cost is constant, model parameters are stationary and the monopolistic firm keeps one product generation
existing in the market at any time. Introducing uncertainty in the model (e.g., Krankel et al., 2006), considering cost learning (e.g., Bass, 1980), allowing model parameters to change over time (e.g., Federguren and Tzur, 1994), inclusion of capacity restrictions (e.g., Haugen et al., 2007), backorders (e.g., Chu and Chung, 2004), customers’ forward-looking behavior (e.g., Shi et al., 2014), incorporating competition in the modeling effort (e.g., Altinkemer and Shen, 2008) and allowing successive generations to coexist during the transitional period between generations (e.g., Jiang and Jain, 2012) offer additional plausible directions for future research. The creation of a less stylized model is a major avenue for future research.

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References


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