Accounting for Banks, Capital Regulation and Risk-Taking*

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Abstract

This paper examines risk-taking incentives in banks under different accounting regimes in presence of capital regulation. In the model the bank jointly determines the capital issuance and investment policy. Given an exogenous minimum capital requirement, lower-of-cost-or-market accounting is the most effective regime that induces the bank to issue more excess equity capital above the minimum required level and implement less risky investment policy. However, the disciplining role of lower-of-cost-or-market accounting may discourage the bank from exerting project discovery effort ex-ante. From the regulator’s perspective, the accounting regime that maximizes the social welfare is determined by a tradeoff between the social cost of capital regulation and the efficiency of the bank’s project discovery efforts. When the former effect dominates, the regulator prefers lower-of-cost-or-market accounting; when the latter effect dominates, the regulator may prefer other regimes.

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1 Introduction

The recent financial crisis has raised a lot of debates about fair value accounting in banks and financial institutions. Advocates for fair value accounting emphasize the benefits in terms of improved transparency and disclosure, promoting market discipline and providing relevant information for decision makers (Landsman, 2005; Ryan, 2009; Laux and Leuz, 2010; and Laux, 2012). Criticisms of fair value accounting mainly focus on the unreliable value estimation for assets with illiquid markets and the systematic risk induced by excessive volatility under fair value accounting (Andrea et al., 2004; Landsman, 2005). Many financial institutions blame fair value accounting for aggravating the financial crisis at a time when markets are extremely illiquid and proper valuation models are unavailable; some even call on the FASB to reassess the new fair value standard.

Given the ongoing debate amid the financial crisis, it is crucial to have a better understanding of the desirability of different accounting regimes for banks so as to provide guidance for policymakers and regulators in the post-crisis regulatory reform. To that end, this paper examines theoretically how different accounting regimes affect the effectiveness of minimum capital regulation in disciplining banks' risk-taking behavior, and how the regulator may optimize the choice of accounting measurements and minimum capital requirements to improve social welfare.

Banks have incentives to engage in excessive risk-taking as a result of high leverage, as shown by Jensen and Meckling (1976). The incentives for risk-taking are greater when banks' investment decisions are not observable or verifiable to outsiders. While debtholders in other industries may protect themselves through various instruments such as covenants and close monitoring, the uninformed small investors with deposits insured by the government lack both the capability and incentives to monitor banks' investment decisions. Therefore banks are subject to prudential regulation where the regulator serves as the representative of small investors (Dewatripont and Tirole, 1994). An important aspect of the current regulatory system is the explicit minimum capital requirement, which was introduced in the Basel Accords as part of the bank regulatory reform in the late 1980s in response to the Savings and Loans (S&L) crisis. By forcing banks to hold more equity capital, it is expected that risk-taking incentives can be reduced. Ideally the inefficiency from...
risk-taking can be eliminated if the regulator requires banks to issue only safe deposits through a sufficiently high capital requirement. However, low levels of deposits are inconsistent with the regulator’s social objective regarding bank’s provision of liquidity services to the economy (John et al. 2000). This paper focuses on the combination of accounting regimes and capital requirements as effective tools for the regulator to achieve the socially optimal investment level while still balancing the liquidity service function of banks.

Whether or not capital requirements can effectively restrict the risk-taking depends crucially on the extent to which the measure of capital is accurate and informative. Therefore capital regulation depends heavily on accounting methods that determine how the net worth (capital) is measured. Three accounting regimes are analyzed in this paper: historical cost accounting (HC), lower-of-cost-or-market accounting (LCM), and fair value accounting (FV). Different accounting regimes affect both the expected earnings to be recognized and the expected regulatory cost of violating the capital requirement. I assume in the model that LCM and FV are equivalent when economic losses are realized; the only difference between these two arises when economic gains are realized.

The basic model follows John et al. (1991), capturing the key feature of banks’ risk-taking incentives in a simple framework. The bank chooses between a safe investment and a risky investment, where the risky investment opportunity only appears after the bank exerts certain effort ex-ante. The project riskiness is privately observable to the bank manager before he makes the investment decision. The bank also simultaneously decides the amount of equity capital to be issued along with the investment policy, and raises the rest of the investment through deposits. The bank’s objective is to maximize a weighted average of the long-run payoff to shareholders and the short-term earnings reported under the prevailing accounting system. This assumption is in line with the myopia literature which typically assumes that managers or current shareholders face and monitoring banks (DeWatripont and Tirole, 1994). In practice, when the capital requirement is violated, it is possible that the regulator may decide whether or not to strictly enforce the capital requirement. For example, during the financial crisis, the regulators may choose not to enforce bank recapitalization to meet the capital requirement. However, such loose enforcement practice also affects the regulator’s reputation and may exaggerate banks’ incentive to take on more risky investments to exploit the regulator’s lack of commitment to regulation. Even though the violation of capital requirement may lead to some inefficiency ex-post, the capital requirement is ex-ante optimal to disciplining the risk-taking incentive in the first place. Morrison and White (2005) demonstrate that a bank regulator actually sets tighter capital requirement than necessary in order to solve the moral hazard problem of “gambling” by undercapitalized banks. The role of capital requirement in reducing risk-taking in banks is modeled in Keeley and Furlong, 1989 and 1990; Rochet, 1992; John et al., 1991.

John et al. (1991) and John et al. (2000) propose other solutions for the regulator to induce optimal risk-taking through either an optimal tax structure or an FDIC insurance premium scheme that incorporates the firm level management compensation schedule in place. These proposals seem appealing in theory, but in practice they are hard to implement. Moreover, both assume that banks equity capital never exceeds the minimum required level, inconsistent with the empirical evidence of excess capital held by many banks.
short-term incentives (Stein, 1989; Narayanan, 1985; and Bebchuk and Stole, 1993). 5

I first consider the problem when the risky investment is always available. Under HC, no information is revealed in the interim period and thus there is no risk of violating the minimum capital requirement ex-post. Therefore the bank will not issue more equity than the minimum capital required and the investment policy will be more risky than the first best policy, the well known risk-shifting problem due to debt financing. Under LCM, the bank may incur a regulatory cost in the face of loss realizations and hence is likely to issue equity capital in excess of the minimum requirement. The optimal investment policy is less risky under LCM than under HC. The bank also issues more capital than the minimum requirement under LCM than HC. FV also helps restrict the bank’s risk-taking behavior similarly to LCM; however, the interest in short-term earnings induces more risk-taking than under LCM.

From the regulator’s perspective, he can always adjust the capital requirement to influence the bank’s capital and investment policy decisions under different accounting regimes. Therefore the regulator’s preference for different accounting regimes depends on the social cost of such a capital requirement (e.g., restricting the liquidity provision function of banks as in Diamond and Rajan, 2000; Gorton and Winton, 1995) and the benefit of reducing ex-post risk-taking. If the capital requirement bears non-negligible social cost, LCM is the most favorable regime while HC is the least favorable. The regulator is able to set lower capital requirement under LCM than other regimes, while achieving the same effect in disciplining the bank’s risk-taking incentive.

When the bank needs to actively exert effort to discover a risky investment opportunity, its ex-ante incentive to do so depends on the benefit from the ex-post risk-shifting. LCM, which is most effective in controlling excessive risk-taking, also most severely discourages ex-ante effort incentives. In this scenario, when the cost of ex-ante effort is non-negligible, the preference of accounting regimes may change. In particular, if the shadow cost of capital requirements is small, LCM may not be socially optimal. When the bank is highly short-term oriented, LCM may make its investment choice too conservative and thereby depress the ex-ante effort incentive.

Finally, I consider several extensions to examine the robustness of main results. First, I consider the mixed-attribute accounting regime in which banks are allowed to report some assets using fair value measurements and some using historical cost measurements simultaneously. The bank’s

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5To the extent that the focus of the paper is the conflict of interests between the shareholders and the debtholders of the bank, I simplify the agency problem between the manager and the shareholders, assuming that the shareholders can design compensation contracts optimally to induce the manager making investment decisions consistent with the shareholders’ objective.
decisions under the mixed-attribute accounting regime appear to be an weighted average of the pure accounting regimes. The more proportion of fair value assets over historical cost assets, the less risk-taking the bank’s investment policy. Secondly, I consider the short-term funding of banks and its implication on banks’ risk taking incentives under different accounting regimes without capital regulation. The short-term funding may also discipline the bank’s risk taking incentive through its interim rollover decision based on informative accounting signals. Therefore, under FV regime the bank’s risk-taking incentive with short-term funding is reduced when compared to the long-term deposit funding. But the short-term funding could not reduce the bank’s risk taking incentive under HC. Lastly, I illustrate two cases of the endogenous cost of violation, in which the bank needs to liquidate some of its existing assets inefficiently, or issue new equity costly when the capital requirement is violated.

These results highlight the importance of incorporating the impact of accounting regimes when bank regulators set the capital rules. After the S&L crisis, the congress required bank regulators to use GAAP as the basis for capital rules. But standard setters of financial reporting have different objectives from the bank regulators and accounting measurements may not provide what the bank regulators desire to use in their regulation. Recently, due to pressure on the accounting standards setting during the financial crisis, the chairman of the FASB (Robert H. Herz) called for the “decoupling” of bank capital rules from normal accounting standards and asked bank regulators to use their own judgement in allowing banks to move away from GAAP. The regulators in fact have deviated from GAAP by being more conservative in measuring banks’ capital. However, it might not be socially efficient to entirely decouple from the accounting standards. The results therefore provide some theoretical basis about the bank regulators’ optimal adjustment of capital requirements according to different accounting regimes in place, and when necessary to deviate from the current GAAP, which direction to go and to what degree.

Another feature of the results in the model is that banks raise more equity capital and implement less risky investment policy when the accounting system is more fair value based (either lower-of-cost-or-market or full fair value accounting). This is consistent with the empirical evidence that banks started to hold more excess capital in 1990s, when the accounting regime moved toward

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4For example, on August 26, 1998, the bank regulators issued a joint final rule that allows banking organizations to include up to 45 percent of net unrealized holding gains on certain available-for-sale equity securities in Tier 2 capital under the agencies’ risk-based capital rules. The rule became effective on October 1, 1998. The full amount of net unrealized gains on such securities are included as a component of equity capital under U.S. generally accepted accounting principles (GAAP), but until the adoption of this rule they were not included in regulatory capital. The full amount of net unrealized losses, in contrast, is deducted from the Tier 1 capital.
a more market-value based system (Flannery and Rangan, 2008). In addition, previous studies on banks typically examine only one aspect of these two decisions, yet the result in this paper demonstrates that the regulator can influence both the level of excess capital and investment risk by adjusting the capital requirement when the accounting regime is not historical-based.

The results have implications on the on-going debates about the pros and cons of fair value accounting in banks and capital regulation post financial crisis (Landsman, 2005; Laux and Leuz, 2009, 2010; Ryan, 2009). Many scholars have argued that blaming fair value accounting exacerbate the financial crisis is unfounded. For example, Laux and Leuz (2010) analyze and review the empirical evidence on the impact of fair-value accounting in the financial crisis and conclude that it is unlikely “fair value accounting contributed to the severity of the financial crisis in a major way”. They suggest that the leverage boom of bank and the reliance on the short-term collateralized repurchase agreement would cause major problems regardless of accounting measurements. The actual impact of fair value accounting on bank’s income and regulatory capital is limited due to the use of discretion allowed in fair value accounting and the mixed attribute accounting system. Ryan (2009) similarly argues that although criticisms about fair value accounting are correct in some aspects, “the subprime crisis that gave rise to the credit crunch was primarily caused by firms, investors, and households making bad operating, investing, and financing decisions, and managing risks poorly”. Kothari and Lester (2012) suggest that accounting for securitization of mortgage-backed securities allows the immediate recognition of gains on securitization may have contributed to aggressive mortgage backed securities related lending, and in addition, the poor implementation of fair value accounting may have been a contributing factor in the crisis. Recent empirical results also provide some evidence consistent with the model’s implications. Ellui, et. al. (2014) show that insurers that employed historical cost accounting engaged in greater degrees of regulatory arbitrage before the crisis and limited loss recognition during the crisis, and insurers with mark-to-market accounting tend to be more prudent in their portfolio allocations. Moreover, Ellui, et. al. (2015) show that insurers facing historical cost accounting are more likely to engage gains trading and less likely to sell downgraded asset-backed securities during the financial crisis.

This paper provides analytical support for these arguments by focusing on the ex-ante role

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7Flannery and Rangan (2008) suggests one reason for the build-up of bank capital is the market discipline force, which is also related to the market-based accounting measurement.

8For example, Keeley and Furlong (1989), Rochet (1992), and John et al. (1991) focus on the bank’s risk choice, taking the capital requirement as exogenous binding constraint; other studies such as Peura and Keppo (2006) focus on the optimal capital decision with capital regulation in the model with costly recapitalization but no risk-shifting incentive.
of accounting in banks' decision making process. The main implication of the model is that a conservative application of fair value accounting (recognizing losses not gains) is most effective in reducing the bank's excessive risk-taking incentives, and that moving to full fair value accounting may not be optimal for disciplining banks. Moreover, disciplining risk-taking may not be the only goal of the bank regulators, and regulators should also balance the bank's liquidity provision function and the benefit from less risk-taking in setting the capital requirement.

The rest of the paper proceeds as follows. Section 2 reviews related literature and institutional background of accounting for banks. Section 3 describes the basic model and assumptions. Section 4 analyzes the bank and regulator's problems under different accounting regimes. Section 5 provides several extensions of the model. Section 6 concludes the paper.

2 Literature and Institutional Background

2.1 Related literature

In a closely related paper, Burkhart and Strausz (2009) study the risk-shifting problem in banks under different accounting regimes but arrive at a different conclusion from this paper. In their model, fair value accounting reduces the degree of asymmetric information asset value, and increases the liquidity of financial assets. As a result the bank's investment opportunities are enlarged and the bank's investment decision is riskier.

Another closely related paper is Bleck and Liu (2007), which also examine the implications of different accounting regimes on the bank's asset price crashes. Bleck and Liu (2007) take the project exogenously given and consider the ex-post liquidation decision of the shareholders when the information about the project state is revealed. They show that lack of transparent accounting information under the historical cost accounting regime delays the efficient liquidation of bad project and results market price crash when the bad project is eventually revealed. In contrast to Bleck and Liu (2007), this paper focuses on the ex-ante investment decision made by the bank under different accounting regimes. We can draw a similar conclusion that historical cost accounting leads to more frequently crashes ex-post when the bank is more likely to undertake risky projects.

There are several theoretical studies about the implications of fair value accounting for financial institutions. These studies mostly adopt an ex-post approach in examining the impact of accounting systems. O'Hara (1993) examines the effect of market value accounting on loan maturity, and finds that market value accounting introduces a bias into the valuation of long-term illiquid assets.
and induces a shift to short-term loans. Freixas and Tsomocos (2004) show historical cost accounting provides better intertemporal smoothing than fair value accounting for banks when dividends depend on profits. More recently, Allen and Carletti (2008) show that mark-to-market accounting can lead to contagion between a banking sector and an insurance sector when the bank’s illiquid assets are carried at the market value, while no contagion would occur under historical cost accounting. Plantin et al. (2008) focus on the problem of fire sales induced by the artificial volatility due to mark-to-market accounting.

More broadly, this paper is also related to prior studies on alternative accounting regimes in other settings. Bachar et al. (1997) compare different accounting valuation approaches in communicating information to investors in a setting with transaction costs and auditing costs. Kirschenheiter (1997) compares the historical cost and market value methods in the valuation of assets. Other papers compare accounting regimes in a contracting setting (Kirschenheiter, 1999) or in a hedge-accounting setting (Melumad et al., 1999 and Gigler et al., 2007). This paper contributes to the literature by comparing accounting regimes in banks and financial institutions and, in the course of doing so, supporting the role of accounting conservatism in financial reporting under certain conditions.

2.2 Accounting for banks

The capital requirements set by Basel committee rely on accounting measures of banks’ capital. Tier 1 Capital mainly includes shareholders’ equity, retained earnings and other disclosed reserves, less goodwill. Tier 2 Capital is the Tier 1 Capital plus provisions or loan-loss reserves, other types of reserves (such as revaluation reserves) and other instruments like subordinated term debt, less the investments in unconsolidated financial subsidiaries and investments in the capital of other financial institution. These capital components are primarily based on GAAP accounting numbers, but bank regulators can make adjustment to elements in the financial statements and define regulatory capital differently from GAAP equity capital.\footnote{For example, for the purpose of calculating regulatory capital, U.S. bank regulators allow most banks the option of excluding accumulated other comprehensive income (AOCI) from the regulatory capital.}

There are two major regimes of accounting measurements that are applied to banks. Fair value accounting requires companies to measure and report assets and liabilities (generally financial instruments) at fair values, which can be measured either at the market price or estimated based on adjusted market prices or valuation models.\footnote{SFAS 157 provides guidelines on measuring fair values and provides a hierarchy of fair value assets based on}

The full fair value accounting means that the
periodic changes of fair values are reported as unrealized gains and loss in the income statement, or in accumulated other comprehensive incomes in shareholders’ equity statement. The alternative to fair value accounting is historical (amortized) cost accounting. Under historical cost accounting, assets and liabilities are reported at historical cost when assets or liabilities were originally generated. Subsequently, historical costs are amortized into income statements. In its pure form of historical cost accounting, all unrealized gains and losses due to subsequent change of asset values are ignored before they are realized. However, assets under amortized cost accounting usually are subject to impairments, which partially adjust the assets’ values to fair value or close to fair value if impairment write-downs are taken. The practice of impairment accounting conforms to the conservatism principle; in the current accounting framework, it is not the same as “one-side” fair value accounting, as the book value of asset is written down only when the impairment is proved to be “other than temporary.”

Accounting in banks and financial institutions has moved toward more fair value based than historical cost based accounting since the savings and loans (S&L) crisis in 1980s. The S&L crisis was in part was attributed to a lack of transparency under historical-cost based accounting (Benston et al., 1986; Furlong and Kwan, 2006). Consistent with the proposal’s recommendation, the use of current valuations among banks and financial institutions has increased over the past 20 years, with FASB’s issuance of a number of accounting standards related to fair value accounting.\textsuperscript{11} However, current accounting practices in banks and other financial institutions still follow a “mixed-attribute” accounting model (Laux and Luez, 2009; Laux, 2012; Nissan and Penman, 2008), in which both fair value and historical cost measurements may be applied to different categories of financial assets and liabilities in the following ways.

- Trading securities by financial institutions (SFAS 115) and certain fair value hedge derivative instruments with fair value option (SFAS 133 and SFAS 159) are reported at fair values on different levels of inputs. Level 1 inputs are observable, quoted prices for identical assets or liabilities in active markets. Level 2 inputs are inputs other than Level 1 quoted prices that are observable, directly or indirectly; examples include quoted prices for similar assets or liabilities in active markets, quoted prices for identical or similar assets or liabilities in markets that are not active, inputs such as observed interest rates, credit risks, volatilities, and default rates, and inputs corroborated by observable market data by correlation or other means. Level 3 inputs are unobservable inputs for the asset or liability, reflecting the firm’s own assumptions about the assumptions that market participants would use in pricing the assets or liability.

\textsuperscript{11}These standards include SFAS 107 (Disclosures about fair values of financial instruments), SFAS 114 (Accounting by creditors for impairment of a loan), SFAS 115 (Accounting for certain investments in debt and equity securities), SFAS 119 (Disclosures about derivatives), SFAS 133 (Accounting for derivative instruments and hedging activities), SFAS 140 (Accounting for transfers and servicing of financial assets and extinguishment of liabilities), SFAS 141 (Accounting and reporting for business combinations), SFAS 157 (Fair value measurements) and SFAS 159 (The fair value option for financial assets and financial liabilities).
the balance sheet and the changes in fair values of these assets (unrealized gains and losses) are included in net income each period.

• Marketable securities classified as “available-for-sale” securities (SFAS 115) and cash flow hedge derivatives (SFAS 133) are reported at fair values on the balance sheet, but the changes in fair values are reported in accumulated other comprehensive income, not in net income. When the change in fair value is deemed as “other-than-temporary”, the loss is recognized in the income statement.

• Other securities classified as “held-to-maturity” securities are reported at historical cost with amortization. However, they are subject to impairment testing every period, and any “other-than-temporary” change in fair value is recognized as loss and the asset is written-down to its fair value.

• Bank traditional loans classified as “held for investment” are valued at historical cost. The “held for investment” loans are also subject to impairment write-downs for any changes in fair values that deemed as “other-than-temporary”.

• Bank traditional loans classified as “held for sale” are reported at the lower of historical cost or fair value (SFAS 65).

• Most liabilities (deposits and debt) are reported at historical cost with amortization.

For most financial institutions, the assets reported at fair value primarily include trading assets and available-for-sale securities, and assets reported at historical cost are primarily loans and “held-to-maturity” securities. For non-fair value assets, typically fair values of these assets are disclosed in the notes to the financial statements according to SFAS 107 and 119. Laux and Luez (2010) report that for large bank holding companies, about 35 percent of assets are reported at or close to fair value; and for investment banks, the fraction of assets reported at fair value tends to be higher. In the mixed-attribute accounting model, the historical cost accounting is not purely historical based, but typically adjusted for the “other than temporary” impairment (OTTIs). Although OTTIs allow the assets value adjusted to fair value for non-temporary asset value loss, it is not exactly the same as pure “lower-of-cost-or-market” accounting. Bank loans classified as “held for sale” are reported at LCM, but as suggested by Laux and Luez (2010), the fraction of bank loans classified as “held for sale” is very small. In terms of bank capital calculation, gains and losses from the change
in fair value for trading securities and OTTIs for other securities affect Tier I Capital directly, but unrealized gains and losses from fair value changes for “available-for-sale” securities are often excluded from the regulatory capital given the option allowed by bank regulators.

To better understand the implications of different accounting regimes in banks, I consider three distinctive measurement regimes: historical cost regime, lower-of-cost-or-market regime, and fair value regime. Each regime is modeled in its pure form, and gains and losses recognized under each regime affect bank capital directly. The practice of accounting for trading and “available-for-sale” securities is consistent with FV regime, and the practice of lower of historical cost or value for “held-for-sale” loans is consistent with the LCM regime modeled in the paper. The practice of OTTI for “held-to-maturity” investment is modified historical cost regime, which reflects some aspect of LCM, if the firm recognizes its fair value losses timely. But in practice, OTTI is implemented with large discretion, and often times the impairment loss is recognized too late to alert investors or regulators about actual changes of bank performance.

In the following section, I first study the bank’s risk-taking incentive under each regime separately. As discussed above, the current accounting practice for banks has the mixed-attribute feature which allows for both historical cost and fair value measurements for different types of assets. Later in the extension section, I extend the main setting to incorporate the mixed-attribute feature in the current accounting practice and discuss the implications of mixed-attribute accounting on banks’ incentives.

3 Model Setup

The basic model is built on the risk-shifting model in John et al. (1991) and John et al. (2000), which captures the key feature of the bank’s moral hazard problem in a simple framework. I first lay out the analytical framework in a general setting and then analyze and compare the bank’s behavior under different accounting regimes.

Consider a three-period model. In period $t = 0$, the bank may exert some effort $a$ to discover a risky investment opportunity. Assume that $a \in [0,1]$ and the cost of effort to the manager is $g(a) = \frac{1}{m}a^2$. The cost is non-pecuniary, and privately incurred by the bank manager. With

\footnote{LCM as modeled in the paper may not be exactly the same as OTTI. Given the noisy accounting signal, if the bank has inside information to prove that the bad news is a false signal about low return in the future, the bank does not need to recognize loss according to OTTI criteria. However, from the investors’ perspective, they value the bank based on the accounting signal, and negative earnings should be recognized upon a bad signal. In the case when the accounting signal is perfect, there is no difference between these two measurements.}
probability \( a \), a risky investment opportunity appears at the beginning of period \( t = 1 \). A safe investment opportunity which generates zero NPV is always available to the bank at the beginning of period \( t = 1 \). Banks can either purely function as an intermediary to provide liquidity and payment services to the depositors, or can be more active in originating commercial loans and other types of investments that can generate value to the economy as a whole. The ex-ante effort in the model could be the bank’s loan origination effort: a bank manager that actively reaches out customers and spends more time to talk to clients to seek opportunities for potential loans is more likely to discover whether such opportunities exist. However, it is the manager’s decision whether or not to make the investment after knowing more about the project’s risk profile. The bank may play an important monitoring role in these risky investment (loans) as in Diamond (1984), and in absence of the bank’s monitoring function, the risky loans may not be profitable to banks. This type of risky investment (loans) requires more expertise and effort from banks in screening and monitoring the borrowers. However, I do not directly model the bank’s monitoring effort in the model.\(^{13}\)

The amount of investment required by either the safe or the risky investment is \( I \). The risky investment generates either high (\( H \)) or low (\( L \)) cash flows at the end of period \( t = 2 \), with \( H > I > L \). The probability of generating the high cash flow \( H \) is \( \tilde{q} \). The probability \( \tilde{q} \) can be interpreted as the credit default risk of the loans that banks originated. Ex-ante all parties know that \( \tilde{q} \) is uniformly distributed over the interval \([0,1]\) and the bank’s effort does not affect the distribution of \( \tilde{q} \). The manager privately observes \( \tilde{q} \) when the risky investment opportunity appears.

The bank’s financing relies on insured deposits and equity issuance. At the beginning of \( t = 1 \), the bank seeks financing by issuing equity \( K \) and collecting deposits of \( D \) to a total amount of \( I \). For simplicity, all deposits are assumed to be insured by the government in the case of default. Thus, the pricing of deposits does not incorporate the default risk of the bank. We can normalize the interest rate of deposits to zero, and the bank promises to pay \( D \) at \( t = 2 \).

The cost of equity is also normalized to zero. There are two reasons to do so. First of all, the cost of equity capital for banks typically is much higher than the cost of deposit financing. Since

\(^{13}\)Banks’ active monitoring may improve the project’s return directly, as in Allen, Carletti and Marquez (2009). In the model, the ex-ante effort \( a \) can also be also interpreted as related to the monitoring role of the bank, and the ex-ante expected return of the risky project increases as \( a \) increases. However, in the model, I do not explicitly model the firm’s investment and borrowing decision from the bank. Rather I assume that the potential profitable loans the bank decide to lend is one with default risk. The main focus of the paper is how accounting measurements affect the bank’s decision in whether or not to invest the risky loans.
banks’ liquidity supply function allows them to easily obtain deposits or various funding sources other than equity capital, the first order determinant of issuing equity is the capital requirement from the regulator. Moreover, shareholders benefit from risk-taking investment from high leverage, hence the shareholders would like to issue as less capital as possible without any capital requirement. Therefore including an explicit cost of equity only complicates the analysis without changing the model’s primary results. Secondly, the model provides a benchmark for equity issuance decision even if we were to consider the cost of equity. If equity is more costly than debt, the bank would like to rely more on debt financing, but wouldn’t be able do so because of capital requirement. But admittedly, the cost of equity capital should also endogenously change when the bank chooses different investment decisions under different accounting regimes, an element that is omitted in the current model. The simplified cost of equity assumption allows us to focus on the first order effects of different accounting regimes on the bank’s investment and capital issuance decision.

I assume that the deposits are long-term deposits to the banks in the sense that they only require to be paid at the end of \( t = 2 \). After the equity and deposits are raised, the bank may choose between the safe and risky projects if the risky investment opportunity appears; otherwise the bank can only invest in the safe project. At the end of \( t = 2 \), the terminal cash flow is realized. The final realized cash flow is observable and verifiable. The bank will default if the realized cash flows is \( L \) and \( D > L \). In the case of default, the government insurance agency will pay depositors the remaining amount of \( D - L \).

The deposit financing in the main model is assumed to be long term. In practice, bank funding comes from a variety of financial instruments other than deposits, especially for non-traditional large banks and growing shadow banks. These banks in the pre-crisis period typically rely on short-term unsecured debt instruments and wholesale funding (through repo market in particular) as their source of funding, which in crisis has caused liquidity risks and bank runs (Raddatz, 2010). In the extension, I study a setup with short-term financing where the accounting information also affects the interim roll-over decision about the short-term debt, and examine how short-term financing under different accounting regimes affects ex-ante risk-taking by banks.

To summarize the model setup, Figure 1 illustrates the timeline of events.

The bank is risk neutral and makes the following investment decision after observing the project’s riskiness \( \bar{q} \).\(^{14}\) Suppose the bank is financed by all equity, then the bank will invest in

\(^{14}\)If the investment decision is made by the manager, who may not be risk neutral, the optimal investment policy will be less risk-taking than what shareholders prefer. But the shareholders optimally design managerial contracts to incentivize the manager taking on the same risk as preferred by the shareholders in order to take advantage of debt.
the risky project as long as the project’s return $\tilde{q}_H + (1 - \tilde{q})L$ is greater than the safe project, i.e.,

$$\tilde{q}_H + (1 - \tilde{q})L \geq I.$$  

Denote $q$ as the lowest cutoff value of riskiness that satisfies the condition, i.e.  

$$q = \frac{I - L}{H - L}.$$  

Since the project’s payoff is increasing in $\tilde{q}$, then the bank invests in the risky project for any $\tilde{q} \geq q$. Therefore the bank’s investment policy is equivalent to a threshold policy.$^{15}$

Definition 1 An investment policy indexed by $q$ is defined as follows: for a given cutoff value of $q$, the bank will choose the risky investment for $\tilde{q} \geq q$ and the safe investment for $\tilde{q} < q$, when the risky investment is available.

Given that $\tilde{q}$ is uniformly distributed over $[0,1]$, an investment policy $q$ produces the following terminal cash flow distribution: $H$ with a probability $\frac{1}{2}(1 - q^2)$, $I$ with a probability $q$, and $L$ with a probability $\frac{1}{2}(1 - q)^2$. The total expected value of terminal cash flows for an investment policy $q$ is thus given by:

$$V(q) = qI + \frac{(1 - q)^2}{2}L + \frac{1 - q^2}{2}H \quad (1)$$

I further assume that $H - I > I - L$, i.e., the risky investment is sufficiently profitable if it succeeds. With this assumption, we can show that the expected project return under the most risky financing. Risk neutrality assumption simplifies the analysis without changing the implications of the model.

$^{15}$A similar argument can be made for the bank’s investment policy when the bank is not fully financed by equity and is subject to capital regulation. Since a higher $\tilde{q}$ always lead to a higher payoff to the bank and a lower likelihood of violating capital requirement, a threshold policy always exists such that as long as it is profitable for the bank to invest in the project at the threshold level, the bank will invest in any other project above this threshold as well.
investment policy \((q = 0)\) is still positive. The first best investment policy \(q^{fb}\), which maximizes \(V(q)\) above, is:

\[
q^{fb} = \frac{I - L}{H - L}
\]  

(2)

The first best investment policy \(q^{fb}\) can be implemented if the bank is financed entirely by equity so that the bank maximizes the firm value, or if the information about \(\tilde{q}\) is perfectly observed by all parties. If the bank is financed through both equity and deposit, the risk-taking problem arises as the bank now maximizes the expected future payoff represented by:

\[
\pi(q, K) = q(I - D) + \frac{1 - q^2}{2} (H - D) - K
\]  

(3)

The optimal investment policy for the bank is \(q^d = \frac{I - D}{H - D}\), which is lower than \(q^{fb}\) if \(D > L\). Therefore as long as the bank issues risky debt, there will be excessive risk-taking in the absence of any regulatory constraint. It should be noted that even if, in contrast to my model’s assumptions, deposit insurance were fairly priced, the risk shifting problem could not be reduced (See Appendix A for the analysis with the fairly priced deposit insurance premium). The reason is that the insurance premium could only reflect the anticipated riskiness of the investment, as the actual realization of \(\tilde{q}\) is privately observed by the bank. The insurance premium only adds a lump sum to the payoff once the equity issued \((K)\) is determined. Excessive risk-taking by banks increases the default probability, which may result in an industry-wide crisis when most banks choose risky investments for individual profit-maximization objectives.

3.1 Information and accounting regimes

Assume that the bank’s private information about \(\tilde{q}\) is not verifiable or contractible. However, the bank has an information system in place that generates signals about the terminal cash flows for the risky investment at the end of the period \(t = 1\). The signal can be either good \((G)\) or bad \((B)\). When the safe investment is chosen, no signal is generated by the information system. The following conditional probabilities represent the properties of the information system.

\[
P(G \mid H) = \alpha
\]

\[
P(B \mid L) = \beta
\]

\(\alpha \in [\frac{1}{2}, 1]\) and \(\beta \in [\frac{1}{2}, 1]\)
When \( \alpha = 1 \) and \( \beta = 1 \), the information system generates perfect signals about the terminal cash flows. Let \( E[V(q) \mid G] \) and \( E[V(q) \mid B] \) denote the expected future cash flows conditional on the signals, given any investment policy \( q \):\(^{16}\)

\[
E[V(q) \mid G] = \frac{\alpha(1-q^2)H + (1-\beta)(1-q)^2L}{\alpha(1-q^2) + (1-\beta)(1-q)^2} \\
E[V(q) \mid B] = \frac{(1-\alpha)(1-q^2)H + \beta(1-q)^2L}{(1-q^2)(1-\alpha) + \beta(1-q)^2}
\]

I also assume for any \( q \leq q^{fb} \), the accounting information system is informative enough that the following conditions always hold, i.e., when the bank takes excessive risk, the expected future payoff to the investment given a bad (good) signal represents a loss (gain):

\[
E[V(q) \mid G] > I; E[V(q) \mid B] < I
\]

Three accounting regimes are considered historical cost accounting, lower-of-cost-or-market accounting, and fair value accounting. Under the pure historical cost accounting, the bank’s assets are carried at the initial investment value \( I \), and there is no earnings recognized before the project’s cash flows are realized. Under the lower-of-cost-or-market accounting, the bank’s assets value are written down when the accounting system reveals bad news about the project’s future cash flows. Under the fair value accounting, the bank’s assets value are either written up or written down when the accounting system reveals good news or bad news about the project’s future cash flows.

Accounting earnings, indicated by \( e_j, j \in h,l,f \), to be recognized at the end of \( t = 1 \) under different accounting regimes, are as follows:

**Historical cost accounting.** No accounting earnings are recognized, i.e: \( e_h = 0 \)

**Lower-of-cost-or-market accounting.** LCM is a form of conservative accounting. The common practice is to write down the book value of the asset to its current market value when the market value falls below the historical book value. When the bad signal is generated, the market value \( E[V(q) \mid B] \) is lower than the book value \( I \) and the bank needs to recognize negative earnings. When the good signal is generated, or no signal is observed, the bank recognizes no earnings. Therefore, accounting earnings under LCM are:

\[\begin{align*}
e_h & = 0 \\
e_l & = 0 \\
e_f & = \frac{\alpha(1-q^2)H + (1-\beta)(1-q)^2L}{\alpha(1-q^2) + (1-\beta)(1-q)^2} \cdot \frac{P(G) - P(L)}{P(G)}
\end{align*}\]

\(^{16}\)Given that \( P(H) = \frac{1}{2}(1-q^2) \), \( P(L) = \frac{1}{2}(1-q)^2 \), and \( P(G) = \alpha \frac{1-q^2}{2} + (1-\beta) \frac{(1-q)^2}{2} \), we can derive (6).
\[ e_l = \begin{cases} 0 & \text{if no (or good) signal is generated} \\ e^B = E[V(q) | B] - I < 0 & \text{if bad signal is generated} \end{cases} \]

*Fair value accounting.* Under FV, the bank has to recognize both the accounting gain (for a good signal) and the accounting loss (for a bad signal):

\[ e_f = \begin{cases} 0 & \text{if no signal is generated} \\ e^G = E[V(q) | G] - I > 0 & \text{if good signal is generated} \\ e^B = E[V(q) | B] - I < 0 & \text{if bad signal is generated} \end{cases} \]

### 3.2 Bank capital and regulation

Besides the investment choice, the bank also endogenously chooses the level of equity capital. I assume that the bank can only issue equity at the beginning of the investment period and the bank’s equity balance is subject to changes due to accounting earnings recognized under different accounting regimes. It is reasonable to assume that the new equity issuance is allowed only at the beginning of the investment period in this setting, as the bank faces the investment opportunity only at the beginning of \( t = 1 \).

From the preceding discussion, it is apparent that without capital requirements the bank will prefer to hold no capital at all. An important element of the current capital regulation is the minimum capital adequacy ratio that the bank needs to meet continually.\(^\text{17}\)

I model the role of this regulatory constraint as follows:

1. At \( t = 0 \), the initial equity issued has to strictly satisfy the capital requirement, which is to hold a minimum capital of \( k \) per unit of deposits.

2. At \( t = 1 \), the bank violates the minimum capital requirement if the new equity balance after recognizing accounting earnings falls below the requirement. The expected regulatory cost of violation is given by the function \( C(u_j(k)) \), where \( u_j(k), \ j \in \{h,f,l\} \), denotes the amount of inadequate capital under respective accounting regimes. If the bank issues equity of \( K \) at \( t = 0 \), then the capital balance at \( t = 1 \) will be \( K + e_j \). The total amount of inadequate capital \( u_j \) at \( t = 1 \) can be represented as:

\(^\text{17}\)The 1988 Basel Accord (Basel I) requires two levels of minimum capital requirements for banks: minimum Tier 1 capital is set at 4% of risk-weighted assets and minimum Tier 2 capital is set at 8% of risk-weighted assets. Banks with at least 5% Tier 1 and 10% Tier 2 capital are considered to be ‘well-capitalized.’ Basel I was replaced by Basel II in 2004. Basel II better aligns the regulatory capital requirements with ‘economic capital’ demanded by investors, which allows the use of the internal ratings based (IRB) approach of choosing regulatory capital.
\[ u_j(k) = \text{Max}\{0, kD - K - e_j\}, \quad j \in \{h, l, f\} \]  

I assume that the cost function is convex, i.e., \( C' \geq 0 \) and \( C'' > 0 \), with \( C'(0) = 0 \). The cost function is assumed to be exogenous, and the regulator can only choose the level of capital requirement \( (k) \) to affect the regulatory cost incurred by the bank.

The cost of violation captures possible negative economic consequences to the bank after violating capital requirement, such as intensified monitoring from the regulator, restrictions on the dividend payments, forced recapitalization, deteriorated credit ratings, and increasing funding cost, etc. It also captures the costly actions that the bank needs to take to avoid violating capital requirement, for example, liquidating some assets to adjust the balance sheet to meet the capital requirement. In the extension, I consider the endogenous cost of violation through inefficient sale of bank’s existing assets before maturity (fire sale) to satisfy the capital requirement.

Empirical studies have documented that many banks hold capital above the minimum regulatory requirement, which is consistent with the model in this paper where the level of buffer capital (i.e., the amount of capital in excess of the regulatory requirement) will be endogenously determined.\(^{18}\)

\(^{18}\)The capital regulation modeled in this paper is consistent with the ex-ante regulation approach in bank capital regulation. Basel I and Pillar I of Basel II are examples of the ex ante capital constraint, which imposes a fixed ratio of the minimum capital requirement. However, Pillar II of Basel II introduces some elements of ex-post regulation, in which the bank has the freedom to choose capital and portfolio risk. This paper does not attempt to model this feature under the new Basel Accord, but it is a possible future direction for research. See Giammarino et al. (1993) and Kupiec and O’Brien (1997) for more details about the ex-post regulation approach.

4 Model Results

4.1 The problems of bank and regulator

4.1.1 The bank’s problem

The bank’s objective function is to maximize a weighted average of short-term earnings recognized and the final expected payoff to shareholders, subject to the cost of capital regulation. The interest in short-term earnings captures the manager’s myopia as opposed to long-term value maximization.
objective (Bebchuk and Stole, 1993; Stein, 1989). Archival evidence shows the importance of short-term earnings to managers, given the pressure from institutional investors and analysts who evaluate company’s performance based on short-term criteria. For example, Graham, et.al. (2005) conduct a comprehensive survey and find that CFOs believe earnings are the key metric considered by outsiders, and that managers trade-off between the short-term need to “deliver” earnings and the long-term objective of making value-maximizing investment decisions. Studies also suggest that shareholders can also be short-termists, who only care about the short-term price they may sell to other investors, instead of long-term value from the company’s investment (Shleifer and Vishney, 1990; Bolton, Scheinkman and Xiong, 2006; Bebchuk, Brav and Jiang, 2015). The bank shareholders with short-term incentives can optimally design managerial compensation contracts to induce the manager making investment decisions consistent with the shareholders’ objective. Therefore I do not explicitly distinguish between the manager and the shareholders’ interest in the investment decision in the model.

To be specific, I assume that the bank assigns some weight, \( \gamma > 0 \), to the earnings reported in the interim period; and the rest \( 1 - \gamma \) to the expected payoff to shareholders, defined as \( \pi(q, K) \) in (3). The expected regulatory cost at the end of \( t = 1 \) is fully internalized by the bank. Hence accounting earnings play a dual role in the model: first, they determine the ex-post cost of violating capital regulation; second, they directly affect the bank’s incentive due to its short-term orientation.

The bank’s expected payoff when choosing the equity issuance and investment policy after the risky investment opportunity appears can be defined as:

\[
\Pi_j(q, K) = \gamma E[e_j] + (1 - \gamma)\pi(q, K) - E[C(u_j)], \quad j \in \{h, l, f\}
\]

Given any minimum capital requirement set by the regulator, the bank’s problem at \( t = 0 \) now

\footnote{Graham, et al. (2005) point out that the managers may report better earnings through either earnings management or the real activities. In this model, I do not consider earnings management through accounting discretions, but the manager can influence the short-term earnings through real activities (i.e., the choices between risky versus safe investments) under different accounting regimes. In the fair value accounting regime, the manager can recognize the gain from the investment before the cash flows are realized, but not in the historical cost or lower-of-cost-or-market accounting regimes. The flexibility to recognize more gains induces the manager to take on more risky investments under fair value accounting regime than the other two regimes.}
becomes:

$$\max_a a \cdot \Pi_j(q^*_j(k), K^*_j(k), k) - g(a)$$

s.t. $$q^*_j(k), K^*_j(k) \in \arg \max_{q,K} \Pi_j(q, K|k)$$

$$D + K = I$$

$$K \geq \hat{K}$$

where $$g(a) = \frac{1}{m}a^2$$ and $$\hat{K} = \frac{k}{k+1}I$$

$$\hat{K}$$ represents the minimum equity capital that satisfies the regulatory requirement at the beginning of $$t = 1$$. The bank chooses the ex-ante effort at $$t = 0$$, and the equity issuance and investment policy after the risky investment opportunity appears to maximize the above objective function under each regime. For any given $$k$$, denote the optimal solution to the bank’s problem above as $$(a^*_j(k), q^*_j(k), K^*_j(k))$$, with $$j \in \{h,l,f\}$$ indicating different accounting regimes.

4.1.2 The regulator’s problem

The regulator can adjust the capital requirement under each accounting regime to maximize his own objective function, which is to maximize social welfare. Notice that the cost to the insurance agency in default is offset by the benefit to shareholders regardless of the insurance premium scheme, and the regulatory cost is a wealth transfer between the regulator and the bank’s shareholders. As mentioned earlier, the regulator can always eliminate the value loss from risk-taking through increasing the capital requirement to the level that the banks are required to issue safe deposits. However, this is not socially efficient as increasing the capital requirement bears certain social costs. One important such cost is the restriction of liquidity creation provided by the bank to investors. Banks’ function as liquidity providers has been extensively studied in the literature following Diamond and Dybvig (1983).\textsuperscript{20} Rather than adopting a full-fledged general equilibrium framework as in these studies, this paper employs a reduced form of the regulator’s objective function to capture both the cost and benefit of imposing capital requirements in a parsimonious fashion. This allows us to concentrate on the impact of accounting regimes.

\textsuperscript{20}For example, Diamond and Rajan (2000) study the consequences of regulatory capital requirements in trading off credit and liquidity creation functions with the possibility of financial distress. Gorton and Winton (1995) also show in a general equilibrium framework to that bank capital is costly because of the restriction on the liquidity provision. Other types of costs associated with capital regulation involve the supervision and compliance costs in general. In a recent study, Van den Heuvel (2008) quantifies the social welfare cost of capital requirements as the percentage of consumption by comparing the benefit of limiting the moral hazard problem and the cost of reducing liquidity creation.
Assume that the economic benefit from liquidity provision can be expressed by some function
\( L(k) \) with \( L'(k) < 0, L''(k) < 0 \) and \( L(0) = 0 \). The regulator’s objective then is to maximize the
following social welfare function under each accounting regime.

\[
\max_k W_j(k) = a_j^*(k)V(q_j^*(k)) - g(a_j^*(k)) + L(k) \\
\text{s.t.} \quad a_j^*(k) \in \arg \max_a a\Pi_j(q_j^*(k), K_j^*(k), k) - g(a) \\
q_j^*(k), K_j^*(k) \in \arg \max_{q, K} \Pi_j(q, K|k)
\]

The cost of violating capital regulation is not explicitly included in the regulator’s objective above. Often times the consequences of violating capital requirement are bank-specific costs, for example, restrictions on dividends payments, increasing funding costs, or intensified monitoring that limit the bank’s investment opportunities. Moreover, these costly actions for bank’s shareholders are typically beneficial to banks’ depositors. As a result, I do not include the cost of violating capital requirement in the regulators’ objective. But it is possible that the violation may also cause some social welfare loss, for example, an ownership change of loans may reduce the value of loans due to inefficient monitoring of the buyer bank. However, it is hard to distinguish between the social loss and the bank specific loss due to the violation of capital requirement. On the other hand, we can interpret the benefit \( L(k) \) as the net benefit of capital regulation, which takes into account the potential social cost of imposing a higher capital requirement. When \( k \) increases, it is more likely to induce capital violation all else equal, and as a result, \( L(k) \) is lower.

The regulator needs to take into consideration both the ex-ante effort and ex-post risk-taking incentives of banks while adjusting the capital requirement. Denote the solution to the above problem under the accounting regime \( j \) as \( k_j^* \). We can compare the social welfare \( (W_j(k_j^*)) \) under different accounting regimes. In the following sections, we are going to analyze the bank’s and regulator’s problems with and without effort incentives, separately.

### 4.2 No ex-ante effort

In this section we analyze a benchmark case when the risky investment opportunity is always available at \( t = 1 \). To be consistent with the model set up, this is equivalent to assuming that the bank’s marginal effort is costless, \( g'(a) \to 0, \forall a \in [0, 1] \). Now the bank’s problem under different accounting regimes is to choose an optimal equity issuance and investment policy at the beginning
of $t = 1$ before the investment opportunity appears. Since the rest of my results crucially depend on the benchmark case without effort incentive, I develop the results in detail in this section.

### 4.2.1 Historical cost accounting

Consider first the historical cost accounting regime. Under HC, no accounting earnings are recognized in the interim period. Therefore the expected regulatory cost is $E[C(u_h)] = 0$ if the initial equity satisfies the minimum capital requirement constraint. Solving the bank’s problem under HC gives the following lemma:

**Lemma 1** Given the exogenous minimum capital requirement $k$, under HC the bank’s optimal investment policy ($q_h^*(k)$) and equity issuance ($K_h^*(k)$) are given by:

$$
q_h^*(k) = \frac{K_h(k)^*}{H - I + K_h^*(k)}
$$

$$
K_h^*(k) = \hat{K}
$$

As the minimum capital requirement ($k$) increases, the bank is forced to raise more capital and will therefore also reduce the riskiness of its investment policy. Another implication of Lemma 1 is that, with historical cost accounting, the bank will issue no more equity than the minimum required level and finance the rest of investment by deposits. In sum, the result is consistent with the prior literature that prudential regulation of banks through minimum capital requirements can reduce the excessive risk-taking by forcing the bank to share the investment’s riskiness to some extent. However, HC fails to capture any new information about the investment’s future cash flows and hence accounting information does not affect the effectiveness of capital requirements in controlling the risk-taking by the bank.

### 4.2.2 Lower-of-cost-or-market accounting

Lower-of-cost-or-market accounting is consistent with the general conservatism principle in GAAP and other accounting standards. Overall, LCM provides more information about the bank’s economic activities, especially when the expected future economic conditions deteriorate. In the model, the accounting system reports a loss of $e^B$ when the bad signal is generated and zero when either the good signal is generated or no signal is generated at all. Therefore the expected earnings to be recognized at $t = 1$ are:

$$
E[e_t] = P(B)e^B
$$

(11)
And the expected regulatory cost is given by:

\[ E[C(u_t)] = P(B)C(u_t|B) \]  \hfill (12)

Ignoring for now the constraint of the minimum equity capital requirement at the beginning of \( t = 1 \), the bank’s optimal choices of equity capital and the investment policy are determined by maximizing the expected payoff \( \Pi_l(q, K) \).\(^{21}\) Denote the solution to this relaxed maximization problem under LCM as \((\hat{q}_l, \hat{K}_l)\). However, the relaxed optimal equity capital \((\hat{K}_l)\) is not always feasible as it may be lower than the minimum capital requirement. Before presenting the complete solution to the bank’s problem under LCM, I also define the minimum capital investment policy as the bank’s optimal risk choice conditional on the initial equity level being equal to the minimum capital required:

**Definition 2** A minimum capital investment policy \((\hat{q}_j)\) is defined as below:

\[
\hat{q}_j \in \max_q \Pi_j(q, \hat{K}), \text{ where } \hat{K} = \frac{k}{k+1}I, \; j \in \{h, l, f\}
\]

Implementing the minimum capital investment policy may not be optimal for the bank, given that the bank also has the option to raise more equity ex-ante to reduce the expected regulatory cost. The bank trades off the marginal benefit and cost of increasing equity beyond the minimum capital requirement level. On the one hand, increasing equity capital reduces the expected future regulatory cost of violating the capital requirement; on the other hand, it reduces the bank’s benefit from risk shifting. This tradeoff depends on the marginal regulatory cost at the minimum capital investment policy level \((\hat{q}_l)\), as demonstrated by the marginal impact of raising additional equity at the minimum capital level:

\[
\frac{\partial}{\partial K} \Pi_l(q, K)|_{\hat{q}_l, \hat{K}} = - (1 - \gamma)^2 \frac{(1 - \hat{q}_l)^2}{2} + (1 + k)P(B)C''(-e^B(\hat{q}_l)) \]

\hfill (13)

**Lemma 2** Given the exogenous minimum capital requirement \( k \), under LCM the bank’s optimal investment choice \((q^*_l)\) and equity issuance \((K^*_l)\) are given by:

- \( K^*_l = \hat{K} \) and \( q^*_l = \hat{q}_l \), when \( C''(-e^B(\hat{q}_l)) \leq (1 - \gamma)^2 \frac{(1 - \hat{q}_l)^2}{2(1 + k)P(B)} \)

\(^{21}\)I show in the proof of Lemma 2 in Appendix B that the interior solution is guaranteed by showing the second order condition is satisfied given the assumption of sufficiently large \( C'' \).
• $K^*_l = \hat{K}_l$ and $q^*_l = \hat{q}_l$, when $C'(\hat{q}_l) > (1 - \gamma) (1 - \hat{q}_l)^2 / 2 (1 + k) P(B)$

Proof. See Appendix. ■

To better understand the intuition behind Lemma 2, note that the optimal investment policy $(q^*_l)$ always satisfies the first order condition $\frac{\partial}{\partial q_l} \Pi_l(q^*_l, K) = 0$ for any level of equity capital $K$. However, the optimal equity issuance decision at $t = 0$ involves a tradeoff between the marginal benefit and cost of increasing equity. Only when the expected marginal regulatory cost is larger than the benefit, will the bank have an incentive to increase its equity capital to the relaxed optimal level (above the minimum capital requirement). Hence one would expect to find banks holding excess capital under LCM, consistent with the empirical evidence that banks started to hold more excess capital in the 1990s when the accounting regime moved toward a more market-value based system (Flannery and Rangan, 2008).

Another observation is that it is never optimal for the bank to issue more equity than $kD - e^B$, i.e., $K > \frac{k}{1 + k} I - e^B$, which is the equity level that fully insures the bank against incurring any regulatory cost. This can be shown following the fact that $\frac{\partial}{\partial K} \Pi_l(q, kD - e^B) < 0$. Given the assumption that $C'(0) = 0$, lowering $K$ slightly starting from $K = \frac{k}{1 + k} I - e^B$, only comes at a second-order loss in terms of expected regulatory costs, while yielding a first-order gain in terms of risk shifting benefits. Thus, instead of holding the capital too safely, the bank will always prefer being exposed to some degree of future regulatory cost.

4.2.3 Fair value accounting

Fair value accounting is a forward-looking accounting regime that requires the recognized asset value to incorporate current information about future cash flows in a fully symmetric fashion. It requires the recognition of both unrealized gains and losses consistently.\(^{22}\) In the context of this model, FV is identical to LCM when there is bad news about future expected cash flows. The only difference between these two regimes arises when there is good news about future cash flows.

Due to the binary nature of the model, the bank has the same expected regulatory cost under

\(^{22}\)SFAS 157 provides an extensive practical guidance regarding how to measure fair values; however, it does not require fair value accounting for any position (Ryan, 2008a). SFAS 159 offers the fair value option to measure certain financial assets and liabilities at fair value, with changes in fair value recognized in current earnings.
both LCM and FV. However, the expected earnings recognized under FV will be higher:

\[ E[e_f] = P(B)e_B + P(G)e_G = \frac{1 - q_f^2}{2}(H - I) + \frac{(1 - q_f)^2}{2}(L - I) \] (14)

Since FV provides full recognition of both gains and losses symmetrically, there is no distortion in the recognized earnings with respect to the expected future cash flows. This transparency property is the main advantage of fair value accounting. Note in particular that the properties of the accounting system do not affect the expected earnings to be recognized under FV.

Similar to the procedure under LCM, I first characterize the relaxed solution to the objective function, ignoring the minimum capital requirement constraint. Again, the relaxed optimal equity issuance is not always feasible as it may be lower than the minimum capital required. Therefore we also need to consider the minimum capital requirement policy under FV, \( \hat{q}_f \), which is derived from the first order condition of the maximization problem of \( \Pi_f(q, K) \). We can characterize the optimal decisions of the bank \( (q_f^*, K_f^*) \) by the relaxed optimal solution and the minimum investment policy under FV. The bank’s issuance of equity capital also depends on two different scenarios under FV, as in Lemma 3:

**Lemma 3** Given the exogenous minimum capital requirement \( k \), under FV the bank’s optimal investment policy \( (q_f^*) \) and equity issuance \( (K_f^*) \) are given by:

- \( K_f^* = \bar{K} \) and \( q_f^* = \hat{q}_f \), when \( C'(-e^B(\hat{q}_f)) \leq (1 - \gamma)\frac{(1 - \hat{q}_f)^2}{2(1+k)P(B)} \)
- \( K_f^* = \bar{K}_f \) and \( q_f^* = \hat{q}_f \), when \( C'(-e^B(\hat{q}_f)) > (1 - \gamma)\frac{(1 - \hat{q}_f)^2}{2(1+k)P(B)} \)

**Proof.** Similar to the proof of Lemma 2 and hence omitted.

After obtaining the optimal solutions under three different accounting regimes, we can compare them and the results are summarized in Proposition 1.

**Proposition 1** Given the exogenous minimum capital requirement \( k \), the bank’s investment policy under FV is less risky than under HC, but more risky than under LCM, i.e., \( q_h^* < q_f^* < q_l^* \); Moreover, the level of equity capital issued under FV is higher than under HC and lower than under LCM, i.e., \( \hat{K} \leq K_f^* \leq K_l^* \)

**Proof.** See Appendix.
Overall both FV and LCM can help to control excessive risk-taking compared to HC. However, FV is less effective in controlling the bank’s risk-taking behavior than LCM. The bank’s concern about the regulatory cost is identical under LCM and FV, but the short-term interest in earnings plays different roles. The short-term interest disciplines the bank in reducing the risk-taking under LCM, while inducing more risk-taking under FV as the upside gain recognized adds the incentive to take on more risk. When the bank puts more emphasis on the interim earnings reported (i.e., $\gamma$ increases), the optimal investment policy under LCM will be less risky. In terms of the equity issuance, the result in Proposition 1 suggests that we are more likely to observe the building of excess capital in banks under LCM than under FV and HC. This is also consistent with the empirical evidence of more excess capital held by banks in 1990s when the accounting system moved toward a more market-value based (mainly the lower-of-cost-or-market value accounting) system after the S&L crisis.

4.2.4 Optimal accounting regime for the regulator

The result above compares the effectiveness of controlling risk-taking under different accounting regimes, holding constant the exogenous minimum capital requirement. However, for the regulator this comparison is not enough to justify which accounting regime is optimal, as the regulator can adjust the capital requirement under each regime to alter the bank’s investment decision. Without the bank’s ex-ante effort incentive, the regulator’s goal is to motivate the bank to choose the socially optimal investment policy $q^{fb}$, assuming the regulator can adjust the capital requirement without any cost. The following lemma then holds:

**Lemma 4** Without ex-ante effort incentive, when capital regulation is costless there exists an optimal minimum capital requirement under each accounting regime, $\bar{k}_j$:

$$\bar{k}_l < \bar{k}_f < \bar{k}_h = \frac{I - L}{L}$$

such that the first best investment policy is always chosen by the bank:

$$V(q^{*}_l(\bar{k}_l)) = V(q^{*}_f(\bar{k}_f)) = V(q^{*}_h(\bar{k}_h)) = V(q^{fb})$$

**Proof.** The proof follows by setting the optimal investment policy $q^{*}_j(k)$ under each regime equal to $q^{fb}$. ■
In this scenario the regulator finds himself indifferent when choosing among three accounting regimes. Under HC, when the capital requirement is \( \tilde{k}_h \), the bank issues only safe deposits as \( D = L \).\(^{23}\) Under the other two accounting regimes, the requirement for issuing safe deposits cannot induce the first best investment policy, as the bank is also subjected to the regulatory cost. The setting in Lemma 4 is clearly unrealistic but serves as a useful benchmark for the following analysis.

In general the optimal capital requirement solving the regulator’s problem will be lower than the level that induces the first best investment policy as in Lemma 4 under each regime. This is because slightly lowering the capital requirement at this level has a positive first order effect on the liquidity provision benefit, while the marginal effect on the investment policy is of second order. When the ex-ante effort incentive is negligible, the bank always exerts the maximum effort level to discover the risky investment opportunity given its benefit from excessive risk-taking. While adjusting the capital requirement, the regulator is only concerned with the tradeoff of inducing the socially optimal investment policy and the cost of capital regulation that limits the bank’s ability to accept more deposits and provide liquidity. The following proposition compares social welfare under different accounting regimes when the regulator optimally chooses the capital requirement accordingly:

**Proposition 2** Without ex-ante effort incentive, social welfare at the optimal capital requirement level is the highest under LCM, and the lowest under HC:

\[
W_l(k^*_l) > W_f(k^*_f) > W_h(k^*_h)
\]

**Proof.** See Appendix.  

The conservative bias under LCM reduces the risk-taking incentives by banks, thereby allowing the regulator to set more lenient capital requirements. This in turn improves social welfare in the presence of the opportunity costs of imposing capital requirements.

### 4.3 Ex-ante effort incentive

The previous section assumes that the bank always faces a risky investment opportunity and then chooses optimal equity and investment decisions at the beginning of period \( t = 1 \) under different accounting regimes. However, often access to the risky investment opportunity is not guaranteed,\(^{24}\) in fact for any capital requirement above this level, the bank's investment policy will also achieve the first best level under HC as the bank internalizes the default risk when only safe deposits are issued.
but depends on the effort exerted by the bank, such as a loan origination process. In this section, I consider the bank’s problem of choosing ex-ante effort to discover a risky investment opportunity. The risky investment opportunity itself is desirable, as the bank can generate positive NPV through investing in the risky project. But under different accounting regimes, the bank’s effort incentive will depend on its anticipated benefit from risk-shifting. Therefore the regulator needs to balance the effectiveness of controlling the bank’s risk-taking behavior and the incentive to motivate the bank to exert effort ex-ante when comparing across different accounting regimes.

Consider the bank’s problem of effort choice under a given accounting regime. The bank will choose between the safe and risky investments as before, if a risky investment is available at \( t = 0 \); otherwise, the bank can only invest in the safe investment. When the bank’s effort and investment choices are not contractible, it chooses the effort level \( (a^*_j) \) in period \( t = 0 \) incorporating its decisions at the beginning of period \( t = 1 \) when the risky investment opportunity appears to maximize its expected utility under each accounting regime. The bank’s period-1 subproblem of choosing equity issuance and investment decisions \( (q^*_j(k), K^*_j(k)) \) under each accounting regime is the same as in Section 4. Solving the bank’s problem in (10) gives the optimal effort level as follows:

\[
a^*_j(k) = \frac{m_1}{2} \Pi_j(q^*_j(k), K^*_j(k))
\]  

Assuming the same level of exogenous capital requirement under different accounting regimes, the bank’s effort increases with its expected payoff from the risky project factoring in its optimal capital and investment policy decisions. We can show that

\[
a^*_f(k) < a^*_l(k) < a^*_h(k)
\]

LCM, which is most effective in disciplining the bank’s risk-taking incentive, also most strongly discourages the bank’s from exerting effort ex-ante to discover a risky project.

Now consider the regulator’s problem when the capital requirement doesn’t bear social cost, \( L'(k) \to 0 \) for any \( k \). The regulator balances both the bank’s ex-ante and ex-post incentives under each accounting regime. Given the same capital requirement, the bank’s effort level under LCM is lower than under the other two regimes. However, the regulator may further lower the capital requirement under LCM to induce higher effort and therefore the comparison of welfare is not clear in this case. In the previous section without the ex-ante effort incentive, the regulator is either indifferent among three regimes or always prefers LCM when the capital regulation is costly,
therefore it is more interesting to look for a scenario when the regulator may prefer other accounting regimes. Proposition 3 below characterizes such a special case:

**Proposition 3** With the ex-ante effort incentive and costless capital regulation, LCM achieves lower welfare than the other two regimes when the bank’s short-term interest is very high ($\gamma \to 1$). Specifically, HC achieves the highest welfare if the bank’s cost for violating capital regulation is very high; otherwise FV achieves the highest welfare.

**Proof.** See Appendix.

Proposition 3 shows that the regulator may prefer HC or FV when the capital regulation is costless but the ex-ante costly project discovery effort on the part of the bank is an important issue. The result is in contrast to Proposition 2, where the regulator always prefers the most conservative accounting regime when the ex-ante effort incentive is not important but the cost of capital regulation is non-negligible.

A comparison of the overall effect on the total welfare becomes intractable when the regulator chooses the optimal capital requirement considering both the ex-ante effort incentive and costly capital regulation. However, we can infer some directions of the prediction from the results in Proposition 2 and 3. When the marginal cost of capital requirement is large and the ex-ante effort incentive is small, the most conservative accounting should be preferred by the regulator. As the ex-ante effort in discovering the investment opportunity becomes more important, conservative accounting will discourage such effort and reduce overall efficiency, therefore might not be the most desirable regime. This result also depends on the degree of myopia in the market. In different economies, the regulators may use their judgement about the bank’s investment opportunity and the market to choose the optimal regime.

## 5 Extensions and Discussions

In this section, I discuss the bank’s problem in several alternative settings. These settings aim to incorporate some key institutional factors that may affect the bank’s risk taking incentives. In the extensions, I focus on discussing the bank’s problem, taking the regulatory capital requirement and the availability of risky investment as given.
5.1 Mixed-attribute accounting

As discussed in Section 2.1, in the current mixed-attribute accounting model, some assets are reported using historical cost measurements, while others are reported using fair value measurements. In this extension, I consider that the bank’s investment in risky assets may be reported either at the historical cost or at the fair value, leaving aside the lower-of-cost-or-market accounting. It is reasonable to assume that when the risky investment opportunity appears at time 1, it may need to be reported following different accounting measurements, but the expected payoff from the risky investment is exactly the same. Assume that with a probability \( m \), the risky investment available involves assets that need to use historical cost measurements, and with a probability \( 1 - m \), the risky investment available involves assets that need to use fair value measurements. All other aspects of the model remain the same as in the main setup.

Now we look at the bank’s optimal investment policy under the mixed attribute accounting for any given capital requirement \( k \). For an investment policy defined by \( q \), the accounting earnings to be recognized at the end of period 1 is either 0 (historical cost) or \( e_f \) (full fair value measurement). Therefore the expected earnings to be recognized at \( t = 1 \) is a weighted average expectation of earnings from both types of assets, i.e.,

\[
E[e_m] = (1 - m)E[e_f] = (1 - m)[\frac{1 - q^2}{2}(H - I) + \frac{(1 - q^2)}{2}(L - I)].
\]

(17)

The amount of inadequate capital \( u_m \) at \( t = 1 \) after earnings are recognized is \( u_m(k) = max\{0, kD - K - e_m\} \). When the investment with historical cost basis appears, \( u_m(k) \) is 0 as long as the initial equity issued satisfies the capital requirement. When the investment with fair value basis appears, \( u_m(k) = u_f(k) = max\{0, kD - K - e_f\} \), which might be non-zero if bad news is reported at time 1. Thus the expected cost of violation under mixed-attribute accounting regime is given by

\[
E[C(u_m)] = (1 - m)P(B)C(u_f|B).
\]

(18)

The bank chooses its optimal investment policy (\( q \)) and the amount of equity issuance (\( K \)) to
maximize its expected payoff as represented in (9) (without effort incentive), i.e.,

\[
\max_{q, K} \gamma E[e_m] + (1 - \gamma)\pi(q, K) - E[C(u_m)]
\]

s.t. \(D + K = I\)
\[
K \geq \hat{K} = \frac{k}{k + 1}I
\]

Solving the above problem following similar approach to the main model, we easily obtain the following result for the mixed-attribute accounting regime.

**Lemma 5** Given the exogenous minimum capital requirement \(k\), the bank’s optimal investment policy \((q_m^*)\) and equity issuance \((K_m^*)\) under the mixed accounting regime are given by:

- \(K_m^* = \hat{K}\) and \(q_m^* = \hat{q}_m\), when \((1 - m)C'(-e^B(\hat{q}_m)) \leq (1 - \gamma) \frac{(1 - \hat{q}_m)^2}{2(1 + k)P(B)}\)

- \(K_m^* = \hat{K}m\) and \(q_m^* = \hat{q}_m\), when \((1 - m)C'(-e^B(\hat{q}_m)) > (1 - \gamma) \frac{(1 - \hat{q}_m)^2}{2(1 + k)P(B)}\)

where \(\hat{q}_m\) is the minimum capital investment policy under the mixed accounting regime as defined in Definition 2. \(\hat{K}m\) and \(\hat{q}_m\) are the optimal equity capital and investment policy without the capital requirement constraint.

**Proof.** Similar to the proof of Lemma 2. \(\blacksquare\)

In the mixed-attribute accounting regime, the bank is also likely to issue buffer equity above the minimum requirement level when the marginal cost of capital violation is relatively high. Comparing the mixed-attribute accounting with the pure accounting regimes, we have the following result:

**Proposition 4** Given the exogenous minimum capital requirement \(k\), the bank’s investment policy under the mixed-attribute accounting regime is less risky than under the pure fair value accounting regime, but more risky than under the pure historical cost accounting regime, i.e., \(q^*_f < q^*_m < q^*_h\);

Moreover, the level of equity capital issued under the mixed-attribute accounting regime is higher than under the historical cost accounting regime and lower than under the fair value accounting regime, i.e., \(K^*_h < K^*_m < K^*_f\).

**Proof.** See Appendix. \(\blacksquare\)

Intuitively, as long as \(m < 1\), i.e., there exists risky investment that needs to be reported using fair value measurement, the mixed-attribute accounting regime reduces the bank’s risk taking incentive compared to the pure historical cost accounting regime. The mixed-attribute accounting
regime leads to an investment policy and equity issuance that appears to be an weighted average of the pure accounting regimes. As the standard setters are debating whether or not to move from the current mixed-attribute accounting to the full fair value accounting regime, the impact on the bank’s risk-taking incentive needs to be taken into accounts.

With mixed-attribute accounting, banks also have more discretion in deciding whether a particular investment (asset) should be classified into the category to be reported using fair value measurements or historical cost measurements. For example, the marketable securities can be classified either as “available-for-sale” or “held-to-maturity”, which are reported using different accounting measurements. The discretion in classification is not considered in the model ($m$ is exogenously given). If we were to consider such a choice, the bank’s decision involves both the ex-ante investment policy and the ex-post classification of the assets. Intuitively, the bank has incentive to classify a risky asset as historical cost basis since it allows the bank to choose ex-ante more risky investment policy (which benefits the shareholders at the expense of debt holders) and incurs lower expected cost of violation of capital requirement, but the fair value basis accounting may have the benefit of increasing the interim earnings reported. The bank has the least incentive to classify the asset as the lower-of-cost-or-market accounting since it only reduces the bank’s risk taking and increases the regulatory capital violation cost, which is consistent with the evidence in Laux and Leuz (2010). Empirical evidence shows that banks overstate the value of distressed assets and regulatory capital during financial crisis using the discretion over the classification of mortgage backed securities (Huizinga and Laeven, 2012). Future research may examine the interaction between the discretion allowed for banks and the rigidity in the regulatory capital requirement.

5.2 Short-term funding

Banks’ funding comes from a wide range of financial instruments, including deposits and non-deposits funding. Before the financial crisis banks (especially for non-commercial banks) increasingly rely on the short-term wholesale funding to supplement deposits funding. The most notable short-term wholesale funding source is the repurchase agreement (repos) and other interbank loans. The short-term wholesale funding enables banks to expand its investment without being constrained by the local deposit funding supply. However, the short-term maturity also increases the bank’s exposure to liquidity risk, which leads to a problem similar to bank runs by depositors as in Diamond and Dybvig (1983). The recent global financial crisis highlights this aspect of negative effect due to banks’ reliance on wholesale funding (Raddatz, 2010).
In this paper, I focus on the bank’s risk-taking incentives on the asset side, leaving aside the liquidity risk problem on the liability side. Compared to the deposit financing, the wholesale funding may perform a function of monitoring or disciplining the bank’s investment due to its market-based nature (Calomiris and Khan, 1991; and Calomiris, 1999). On the negative side, Huang and Ratnovski (2011) point out that the wholesale funds with collateral feature may have an incentive to withdraw funding on the basis of a noisy signal of bank asset quality, which induces socially inefficient liquidation of banks’ assets. Since accounting information provides updated signals about the bank’s performance to the short-term funding market, it is interesting to examine how the short-term funding affects the bank’s ex-ante risk-taking incentive under different accounting regimes. I will discuss how the short-term funding affects the bank’s risk-taking incentive in the basic setup without the capital requirement and the myopic incentive.

In the model setup (Section 3), we have introduced bank’s problem when financed with the long-term deposit in equation (3) as below:

$$
\Pi(q, D) = q(I - D) + \frac{1 - q^2}{2}(H - D) - K.
$$

In the problem, the deposit $D$ issued at date 0 is long-term debt and repaid only at the end of the period 2 when the actual cash flows are realized.

Now assume that instead the bank seeks financing from the short-term funding market (for example, Repo market) with an amount of $D^s$, and it matures at date 1 with a repayment of $D^1$ at date 1. At date 1, the short-term fund lender decides whether or not to rollover the debt to next period, and if rollover is successful, the bank pays $D^2$ at the end of period 2. If the rollover is not granted, the bank needs to liquidate its assets and terminate (in our model) its project to meet the debt payment and collateral requirement. For simplicity, I assume that the interest rate of short-term debt is determined by the market rate, which does not depend on the bank’s own performance. Furthermore, such interest rate is normalized to zero in the model. Therefore with zero expected return, $D^2 = D^1 = D^s$ if the rollover is granted. With the short-term funding, the interim accounting signals play an important role in the rollover decision of short-term funds.

Under historical cost accounting, no information is revealed at date 1 and the market’s perception about the bank’s performance does not change from date 0. Therefore the short-term debt is

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24 The assumption is consistent with the fact that Repo market provides cheap funding to banks with a collateral agreement and its interest rate is often lower than the federal funds. Due to the collateral agreement, the bank’s investment is subject to liquidation if the market perceives negative news about the bank’s future cash flows.
rolled over automatically in absence of any information. Hence, under historical cost accounting, the bank’s incentive problem does not change at all compared to the long-term deposit financing. The optimal investment policy for the bank is \( q_h^* = \frac{I - D_s}{H - D_s} \), which is exactly the same as the bank’s optimal investment policy with the long-term deposit financing.

Under fair value accounting, when accounting system generates good signal, the short-term debt is rolled over. When accounting system generates bad signal, the market does not rollover the debt and demands collateral collection. Thus the bank needs to liquidate its project early, and the bank receives nothing from liquidation. After the bank privately observes the realized probability of generating the high cash flow \( \tilde{q} \), the bank makes a decision whether or not to invest in the risky project, taking into account the consequences of risky investment under fair value accounting. The bank receives payoff at the end of period 2 only when the project generates a good signal and the project succeeds with high realized cash flows. Thus the bank’s expected payoff from investing in the risky project (\( \tilde{q} \)) is

\[
\text{Prob}(G)P(H|G)(H - D^s) = \alpha \tilde{q}(H - D^s).
\]

The payoff in risky project is nondecreasing in \( \tilde{q} \). Therefore the bank’s investment policy must follow a threshold policy, i.e., there exists a threshold \( q_s^* \), such that the bank invests in the risky project whenever \( \tilde{q} \geq q_s^* \), and invests in the safe project otherwise. Therefore the bank’s expected payoff following an investment policy of \( q_s^* \) and short-term financing of \( D^s \) is given by:

\[
\Pi(D^s, q_s^*) = q_s^*(I - D^s) + E_{\tilde{q} > q_s^*}[\alpha \tilde{q}(H - D^s)].
\]

Taking the first order derivative of the expected payoff with respect to \( q_s^* \), we get

\[
(I - D^s) - \alpha q_s^*(H - D^s) = 0.
\]

Solving the first order condition above, we obtain the optimal investment policy with short-term financing under fair value accounting regime, \( q_f^* = \frac{(I - D^s)}{\alpha(H - D^s)} \). Compare with the optimal investment policy under historical cost accounting regime, we easily derive the following proposition:

**Proposition 5** With the short-term financing, the bank’s investment policy is less risk-taking under fair value accounting regime than under historical cost accounting regime, i.e., \( q_f^* \geq q_h^* \).

The intuition is straightforward. Without the capital requirement, the short-term funding may also discipline the bank’s risk-taking incentive through the interim rollover decision under fair value accounting regime. If the bank’s project is liquidated early, the bank receives zero payoff. Such
disciplinary role of short-term funding occurs only when the market receives timely bad signals about the bank’s project outcome under fair value accounting regime. It is easy to infer that lower-of-cost-or-market accounting provides the same disciplinary effect as fair value accounting, since the bad signal is also revealed to market under LCM and results early liquidation of the project.

Moreover, it is interesting to observe that only an imperfect accounting signal can discipline the bank. The bank’s incentive to invest in the risky project is lower because an imperfect accounting signal may induce inefficient liquidation of the project with high cash flows. When $\alpha = 1$, the bank’s investment policy is exactly the same as under historical cost accounting. The bank’s incentive in risk-taking decreases as the accounting signal becomes less precise ($\alpha$ decreases). However, this does not suggest that a completely uninformative accounting system is the best. If the accounting signal is not informative at all, the market will not rely on accounting signals to make the rollover decision, similar to Huang and Ratnovski (2011). As assumed in the main model, as long as the accounting system is sufficiently informative, the short-term debt is rolled over upon a good signal and the bank’s investment policy is less risk-taking than under historical cost accounting. But further increasing the informativeness of accounting system ($\alpha$) above the minimum informativeness level only increases the bank’s risk-taking incentive. If the standard setters may influence the information quality of accounting system, the result here suggests that some degree of opacity might be optimal for disciplining banks financed by short-term fundings.

Empirical evidence shows that banks funded by traditional deposits and long-term funding are less risky before the financial crisis (Demirguc-Kunt and Huizinga, 2010). The discussion above suggests that short-term funding per se is not the cause of excessive risk-taking in banks. In fact, the short-term financing may discipline the risk-taking incentive better if fair value accounting is applied. However, before financial crisis, the availability of short-term funding enlarges the bank’s investment opportunities, which naturally results more risk-taking by banks due to a higher level of debt financing. In addition, another important factor unaddressed in my model is the securitization of mortgage backed securities. Accounting standards allow the securitization to be recognized as sale, which essentially keeps these mortgage assets off-balance sheet, and reduce the capital requirement for the risky assets invested. The securitization related question is not discussed in this paper, but future research may examine how the securitization and accounting for securitization affect the bank’s risk-taking incentives before and during the financial crisis.
5.3 Endogenous cost of violation

In the main setup, the bank’s cost of violating capital requirement is assumed to be an exogenous function of the insufficient capital. The cost of violation reflects either the regulatory cost directly imposed by the regulator (such as restrictions on dividend payments, or subsequent intensified monitoring), or the costly actions that the bank may need to incur to avoid the capital regulation. In this extension, I provide the analysis with endogenous cost examples through two channels: 1) inefficient liquidation of existing assets; and 2) costly equity issuance after the violation. This question is irrelevant for historical cost accounting, since no violation of capital regulation is expected to incur under historical cost accounting regime. For simplicity, I only consider the bank’s problem under fair value accounting regime in this section. The lower-of-cost-or-market accounting has exactly the same effect as fair value accounting since the costly liquidation of assets is relevant only when the bad news is recognized.

5.3.1 Inefficient asset liquidation

To introduce the costly liquidation in the model, I assume that the bank has an existing asset of $A_0$, an existing debt of $D_0$ and existing capital of $K_0$ before making the new investment decision to expand the bank’s balance sheet. $A_0 = D_0 + K_0$, and furthermore, the existing capital satisfies the binding constraint of the capital requirement, i.e., $K_0 = kD_0$. The new investment opportunities and new equity issuance follow the assumptions in the main model. At date 0, the bank’s balance sheet after investing in the new project is given in Table 1a. At date 1, if accounting earnings are recognized, the bank’s balance sheet after earnings before the sale of any asset is presented in Table 1b.

At date 0, the total equity capital ($K + K_0$) satisfies the capital requirement. At date 1, if negative earnings ($e^B < 0$) are recognized upon a bad signal, the total capital becomes $K + K_0 + e^B$, and the total assets becomes $A_0 + I + e^B$. The bank’s capital ratio might fall below the minimum capital requirement if the initial capital buffer is not high enough to cover the earnings recognized, i.e., $K + K_0 + e^B < k(D_0 + D)$. The bank may consider sell some of its existing assets ($A_0$ is assumed to be separable) to satisfy the capital requirement. The sale of asset before maturity results a loss of asset value. Assume that the market price of the asset sold is discounted by a factor $\delta$, and that the discount factor depends on the amount of insufficient capital $u$ at date 1, $u = max\{kD - K - e_f, 0\}$. Furthermore, assume that $\delta(u)' > 0$ and $\delta(u)'' > 0$.\(^{25}\) The discount

\(^{25}\)The assumption of convexity with respect of insufficient capital only serves to insure an interior solution available
factor may reflect either the loss of fundamental value of asset due to change of ownership (for example, the new buyer may not have the same sophisticated skills in monitoring the loans as the current bank), or the liquidity pricing discount of the asset when the capacity of the loan sale market is limited. It is reasonable to assume that the more the bank needs to sell its asset for (a higher $u$), the more discount is applied to the sale price.

Now consider the bank sells $\phi \in [0, 1]$ proportion of its current asset and receives $(1 - \delta(u))\phi A_0$. The cash received can be used to payoff some of its current debt so that the capital requirement is still satisfied. Since the asset is sold at a discount compared to the book value, a loss of $\delta(u)\phi A_0$ is recognized. The bank's balance sheet after partial sale of asset is presented in Table 1c.

Table 1: Bank’s Balance Sheet

<table>
<thead>
<tr>
<th></th>
<th>a. Date 0</th>
<th>b. Date 1</th>
<th>c. Date 1 (after sale)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$D$</td>
<td>$D_{1+e_f}$</td>
<td>$D_{1+e_f}$</td>
</tr>
<tr>
<td>K</td>
<td>$K_0$</td>
<td>$K_{1+e_f}$</td>
<td>$K_{1+e_f}$</td>
</tr>
<tr>
<td>$A_0$</td>
<td>$D_0$</td>
<td>$A_0$</td>
<td>$(1 - \phi)A_0$</td>
</tr>
<tr>
<td>$K_0$</td>
<td>$K_0$</td>
<td>$K_0$</td>
<td>$K_0 - \delta\phi A_0$</td>
</tr>
</tbody>
</table>

The proportion of assets to be sold needs to satisfy the following condition:

$$K_0 + K + e^B - \delta(u)\phi A_0 \geq k(D_0 + D - (1 - \delta(u))\phi A_0)$$

$$\Rightarrow \phi \geq \phi^* = \frac{u}{A_0[k(1 - \delta(u)) - \delta(u)]}$$

It is easy to see that the minimum portion of assets to be sold ($\phi^*$) is decreasing in the equity capital issued $K$ for any given level of capital requirement. Now we consider the bank’s choice of investment policy and equity issuance at date 0 to maximize its expected payoff as specified below:

$$\max_{q,K} \gamma E[e_f] + (1 - \gamma)\pi(q, K) - E[\phi^*\delta(u)A_0]$$

s.t. $D + K = I$

$$K > \frac{k}{k + 1}I$$

The liquidation induces an expected loss of the bank’s current asset value and is incorporated in the bank’s objective function. Solving the problem above, we obtain the following proposition:

for the optimal equity issuance decision. If instead we assume that $\delta$ is independent of $u$, we can show that when $\delta$ is below a threshold, the bank always issues $K = \hat{K}$; when $\delta$ is above the threshold, the bank always issues capital sufficiently high, $K = \hat{K} + e_B$, so as to completely avoid the need for costly liquidation of assets. In both cases, the investment policies under fair value accounting are still less risky than under historical cost accounting.
Proposition 6 When the bank needs to sell some existing assets to satisfy the capital requirement, the bank’s optimal investment policy under fair value accounting is less risky than under the historical cost accounting regime, and the bank issues a higher than minimum level of equity capital under fair value accounting. i.e., \( q_f^* > q_h^* \) and \( K_f^* \geq K_h^* = \hat{K} \).

Proof. See Appendix. ■

The bank may adjust the balance sheet by selling some assets when the capital requirement is about to be violated, but such liquidation comes at a cost. Under fair value accounting, a bank that invests in more risky project is expected to incur a greater loss in asset liquidation. Therefore the bank’s incentive to take on more risky projects is reduced under fair value accounting.\(^{26}\)

Since the inefficient liquidation of assets is also socially costly, the regulator may also internalize this loss in his welfare function. That is, in the objective function in (21), the expected payoff from the bank’s investment becomes \( V(q_j(k)) - E[\phi^* \delta(u) A_0] \). Incorporating the asset liquidation loss does not affect our main result about the regulator’s optimal choice of minimum capital requirement. To see this, we recall that the bank needs to inefficiently liquidate a fraction, \( \phi^* \equiv \frac{u}{A_0(1 - \delta(u))} \), of existing asset to meet the capital requirement. For a given amount of insufficient capital \( u \), a higher capital requirement reduces the required fraction of inefficient sale. The reason is that liquidating the existing asset to pay off the old debt essentially releases certain amount of equity capital from the required capital, which is given by \( k(1 - \delta)\phi A_0 \). It is easy to see that the higher the capital requirement, the more equity capital is released from paying off the old debt, and the released capital makes up the insufficient capital due to the loss recognized from the new investment at date 1. Therefore increasing minimum capital requirement \( k \) not only reduces the risk-taking in investment policy, but also reduces the expected loss from the future inefficient asset liquidation. The regulator has incentive to increase the minimum capital requirement that maximizes the project’s total payoff. However, considering the aforementioned social cost of liquidity provision by increasing the capital requirement, we again reach a similar conclusion about the regulator’s preference as before.

\(^{26}\)Notice that there is an implicit upper-bound for the discount factor in asset sale, in order for the sale of asset to be an effective way to resolve the insufficient capital problem. From (21), the proportion of assets to be liquidated needs to be positive, i.e., \( k - \delta(u)(1 + k) > 0 \Rightarrow \delta(u) < \frac{k}{1 + k} \). Otherwise, the loss from inefficient sale of asset might be too big to restore the capital requirement of the bank. This may have some implications for the bank’s fire sale during the financial crisis. When the bank’s minimum capital requirement is breached, the fire sale at the time of a huge discount may not help the bank to meet the capital requirement, but only induces inefficient loss of asset value. However, the bank may still need to incur fire sale of assets in order to satisfy the liquidity need of the bank. The liquidity risk and the regulation of liquidity requirement, though beyond the scope of this paper, belong to another aspect of bank regulation of huge importance for banks.
5.3.2 Costly equity issuance

The bank may issue new equity at date 1 when the capital requirement is violated. The new equity issuance at this time can be costly to the equity investors. To illustrate this example, consider a similar setup as the main model, and the bank decides the initial equity capital issuance \( K \) which satisfies the capital requirement, and chooses the investment policy \( q \) at date 0. At date 1, the bank may need to issue additional equity capital \( K_1 \) if the total capital falls below the minimum capital requirement after recognizing negative earnings upon a bad signal. The new capital issuance amount should be at least the amount of insufficient capital, i.e., \( K_1 \geq u \), where \( u = \max\{kD - K - ef, 0\} \).

For simplicity, assume that the new capital raised is invested in the safe asset, which generates the same amount of cash flows of \( K_1 \) at date 2. However, for the equity investors, they actually receive less than the entire payoff from the new investment, due to the existing first period risky investment. When the first period’s risky project generates a low cash flow \( L \), the equity holders receive \( \max\{0, L + K_1 - D\} \). That is, the cash flows from new investment is paid to the deposit holders first if the risky project’s payoff is not sufficient to cover the deposit amount. As a result, the equity investors implicitly commit additional payment to the depositors for the risky project’s low outcome by issuing new equity in the interim stage.\(^{27}\) The new equity issuance at date 1 is therefore costly to the equity investors. The expected loss is given by

\[
P(L|B)(\max\{0, L + K_1 - D\} - K_1)
\]

Formally, the bank solves the following problem when incorporating the future expected loss from equity issuance at date 1,

\[
\max_{q, K, K_1} \gamma E[e_f] + (1 - \gamma)[\pi(q, K) + P(L|B)(\max\{0, L + K_1 - D\} - K_1)]
\]

s.t. \( D + K = I \)

\( K > \frac{k}{k + 1} I \)

\( K_1 \geq u, \text{ where } u = \max\{kD - K - ef, 0\} \)

\(^{27}\)Even if the new investment at date 1 has positive NPV, we still face a similar problem in the sense that equity investors receive less payoff from the new investment when the existing risky investment is partially financed by debt. This is a typically debt overhang problem in corporate finance. In our model, due to the minimum capital requirement, the bank has to issue new equity and incur the potential loss. However, the bank’s incentive to make new investment would be lessened without the regulatory requirement.
It is easy to infer that the equity investors do not have incentive to issue more than the amount of insufficient capital $u$ at date 1, i.e., $K_{1}^{*} = u$. Similar to the costly asset liquidation case, the violation cost function $c(u)$ in the main setup is now represented by the expected loss from future equity issuance. Therefore we obtain a similar result as in the main setup.

**Proposition 7** When the bank expects to issue new equity in the future to satisfy the capital requirement when it is violated, the bank’s initial investment policy under fair value accounting is less risky than under the historical cost accounting regime, and the bank also issues a higher initial equity capital than the minimum requirement under fair value accounting. i.e., $q_{f}^{*} > q_{h}^{*}$ and $K_{f}^{*} > K_{h}^{*} = \hat{K}$.

Since the expected loss from future equity is linear in the insufficient capital at date 1, the bank either issues the minimum amount of capital or issues a sufficient amount of capital at the initial date in this setup. When the bank issues a sufficiently high initial capital ($K^{*} = kD - P(B)e^{R}$), which completely avoids the loss from the costly future equity issuance, the bank also chooses a less risky investment policy because of the higher equity stake in the project. When the bank issues the minimum amount of capital at the initial date ($K = \hat{K}$), the bank’s investment policy is still less risky than under the historical cost accounting because of the expected loss in future equity issuance. Therefore, fair value accounting disciplines the bank’s risk-taking incentive at the initial stage.

6 Conclusion

This paper examines banks’ risk-taking incentives in the presence of minimum capital regulation under three different accounting regimes: HC, LCM and FV. LCM, which requires banks to recognize economic losses earlier when information becomes known to the market, is shown to be more effective than the other two regimes in controlling risk-taking behaviors by banks. Moreover, banks are more likely to hold buffer capital to avoid future costly violation of capital regulation when the accounting system incorporates more market-based information. Compared to LCM, FV may be less effective in controlling risk-taking, because recognizing positive news gives banks additional incentives to be more aggressive ex-ante in risk-taking when banks care about short-term earnings recognized in addition to the expected final payoff to shareholders.

When the regulator may adjust the minimum capital requirement optimally under each accounting regime, social welfare is the highest under LCM and the lowest under HC if increasing
the capital requirement also increases the social cost. On the other hand, when the role of ex-ante effort by the bank in discovering the investment opportunity is more important, I show that the above preference order may reverse if the bank is sufficiently short-term oriented.

Admittedly, the implementation of fair value accounting (or the lower-of-cost-or-market accounting) faces challenges and the current bank accounting measurement rules leave much discretion for banks in choosing whether or not to apply fair value measurements to certain assets, and if applied, what models to implement in measuring the fair value. From the bank regulator’s perspective, the capital regulation needs to take into account the bank’s accounting rules and measurement problems, and if necessary, decouple the regulatory capital calculation from the full fair value basis in order to prudentially regulate the bank’s excessive risk-taking which eventually leads to crisis.

One relevant concern for standard setters is the recognition versus disclosure of fair value. The model in this paper has two implications. First, for effective capital regulation, recognition of economic losses is essential to get an accurate measure of the capital; disclosure of fair value itself cannot bring the declining economic value of banks’ capital to the regulator’s attention. Second, the short-term interest in earnings likely depends on the market’s reaction to accounting information. Given that the degree of market reaction is larger for recognized earnings than for disclosed numbers, the recognition of upside gains may induce more risk-taking by banks than pure disclosure; however, the recognition of downside losses can better discipline risk-taking as shown in the model. Therefore, this paper suggests that LCM with disclosure of full fair value is a better combination for the accounting framework in banks.
References


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Appendix A: Fairly priced deposit insurance

In the model, I assume that deposits are fully insured by the insurance agency and the insurance agency may demand an insurance premium from the bank for each dollar of deposit raised. Will the fairly priced (risk-sensitive) deposit insurance premium solve the problem of risk taking? In the analysis of the main text, the bank’s payment for the insurance premium is not included in the objective function. The following analysis explains why the bank’s optimal decisions are not altered by the existence of a fairly priced insurance system, even if the bank incorporates the insurance premium cost in the objective function.

In this appendix, I analyze the bank’s problem in Lemma 1 considering fairly priced deposit insurance. Suppose that the insurance agency now prices the insurance of deposits $D$ based on the expected default cost when the bank chooses its investment policy of $q$. A fairly priced insurance premium is specified as follows:

$$p(D, q) = \frac{(1 - q)^2}{2} (D - L)$$  

(24)

Ideally, if the bank internalizes the insurance cost in the objective function, the bank faces the problem as stated below:

$$\max_q \pi_h(q) = q(I - D) + \frac{1 - q^2}{2} (H - D) - p(D, q) - K$$

The investment policy that solves the above problem is $\frac{I - L}{H - L}$, which equals the first best investment choice $q^{fb}$. However, since the bank’s investment riskiness is not observable to the regulator, the regulator cannot enforce or monitor the bank’s investment decision once deposits are raised. If the bank issues deposits with the insurance premium priced as $p(D, q^{fb})$, it will always have the incentive to deviate from $q^{fb}$ so as to maximize the expected payoff in the following equation:

$$\max_q \pi_h(q) = q(I - D) + \frac{1 - q^2}{2} (H - D) - p(D, q^{fb}) - K$$

Then the optimal solution to the above problem is given by $q^* = \frac{I - D}{H - D}$, which yields the same investment policy as in Lemma 1. Essentially the risk-shifting problem of the bank in my model is driven by the incomplete contractable investment choice, which can not be solved through the fairly priced insurance premium.

The insurance agency can, nonetheless, still set a fairly priced insurance premium based on the predicted bank’s optimal decisions under different accounting regimes. As specified below, the insurance premium depends on the capital structure and the anticipated investment policy of the bank:
\[ p(D, q_j^*) = \frac{(1 - q_j^*)^2}{2}(D - L), \text{ where } j \in \{h, l, f\} \]  

(25)

With the fairly priced insurance premium, the bank’s shareholders actually pay the cost of the sub-optimal investment choice induced by the deposit financing. The bank’s investment riskiness can only be controlled through effective capital regulation or other mechanisms not examined in this paper.

**Appendix B: Proof**

**Proof. Lemma 2**

To solve the objective function in (9) subject to the capital requirement constraint without ex-ante effort incentive, we first look at the case when \( u = 0 \), i.e., \( K \geq kD - e^B \). Since \( e^B < 0 \), \( K \geq kD \) is automatically satisfied. Now the objective function becomes:

\[ \Pi_l(q, K) = \gamma P(B)e^B + (1 - \gamma)[qK + \frac{1 - q^2}{2}(H - I + K) - K] \]

Taking the first order derivative with respect to \( K \), we get:

\[ \frac{\partial}{\partial K} \Pi_l(q, K) = -(1 - \gamma)(1 - q)^2 < 0 \]

Hence it is never optimal to issue equity more than \( kD - e^B \).

Now we examine the case when the equity issuance level is less than \( kD - e^B \):

If \( K < kD - e^B \), \( u = kD - K - e^B > 0 \), the objective function becomes:

\[ \Pi_l(q, K) = \gamma P(B)e^B + (1 - \gamma)[qK + \frac{1 - q^2}{2}(H - I + K) - K] - P(B)C(kD - K - e^B) \]

Taking the first order derivative of the above function with respect to \( K \) and \( q \), we get:

\[ \frac{\partial}{\partial K} \Pi_l(q, K) = -(1 - \gamma)(1 - q^2) + P(B)(1 + k)C'(u) \]

\[ \frac{\partial}{\partial q} \Pi_l(q, K) = \gamma[\beta(1 - q)(I - L) - q(1 - \alpha)(H - I)] + (1 - \gamma)[K - q(H - I + K)] - \frac{\partial P(B)}{\partial q}C(u) + P(B)C'(u)\frac{\partial e^B}{\partial q} \]

The second order derivatives and cross partial derivative with respect to \( K \) and \( q \) are given by:
\[
\frac{\partial^2}{\partial K^2} \Pi_l(q, K) = -P(B)(1 + k)C''(kD - K - eB) < 0
\]
\[
\frac{\partial^2}{\partial q^2} \Pi_l(q, K) = \gamma \left[ (1 - \alpha)(H - I) - \beta(I - L) \right] - (1 - \gamma)(H - I + K) < 0
\]

By assumption (7):
\[
\frac{\partial^2 P(B)}{\partial q^2} C(u) < 0 \quad \frac{\partial^2 P(B)}{\partial q^2} C'(u) < 0 \quad \frac{\partial eB}{\partial q} > 0
\]
\[
\frac{\partial eB}{\partial q} < 0
\]
\[
\Rightarrow \frac{\partial^2}{\partial q^2} \Pi_l(q, K) < 0
\]

Check whether the Hessian Matrix is positive definite, i.e.,
\[
\frac{\partial^2 \Pi_l}{\partial K^2} \frac{\partial^2 \Pi_l}{\partial q^2} - \left( \frac{\partial^2 \Pi_l}{\partial K \partial q} \right)^2 > 0
\]

It turns out that as long as the \( C'' \) is sufficiently large, the above condition always holds. Therefore the second order condition for maximizing \( \Pi_l(q, K) \) without the second constraint is satisfied.

Now define the solution that satisfies the first order condition as \( \hat{q}^c_{lcm} \) and \( \hat{K}^c_{lcm} \), which are given as follows:

\[
\left\{ \begin{array}{l}
\frac{\partial \Pi_l(\hat{q}_l, \hat{K}_l)}{\partial q} = 0 \\
\frac{\partial \Pi_l(\hat{q}_l, \hat{K}_l)}{\partial K} = 0
\end{array} \right.
\]

Considering the capital requirement constraint that \( K \geq kD \), let \( \hat{K} = kD \). The following scenarios are considered:

- If \( \frac{\partial \Pi_l(\hat{q}_l, \hat{K}_l)}{\partial K} \leq 0 \), i.e., \( P(B)(1 + k)C'(eB) \leq (1 - \gamma) \frac{(1 - \hat{q})^2}{2} \), then the bank would want to further decrease the equity capital to the relaxed optimal \( K \) which is below the capital requirement level, but couldn’t do so because of the capital requirement. Hence, given the capital requirement constraint the bank’s optimal equity level is \( K^*_l = \hat{K} \).

- If \( \frac{\partial \Pi_l(\hat{q}_l, \hat{K}_l)}{\partial K} > 0 \), i.e., \( P(B)(1 + k)C'(eB) \geq (1 - \gamma) \frac{(1 - \hat{q})^2}{2} \), then the bank could further increase the equity capital to the relaxed optimal level, which is \( K^*_l = \hat{K} \).

Given the optimal level of the equity capital \( K^*_l \), the optimal investment policy \( q^*_l \) is always determined by the first order condition \( \frac{\partial}{\partial q} \Pi_l(q, K^*_l) = 0 \). Therefore, when \( K^*_l = \hat{K} \), it is the
minimum capital investment policy \( q_i^* = \hat{q}_i \); when \( K_i^* = \hat{K}_i \), it is the relaxed optimal investment policy \( q_i^* = \hat{q}_i \).

Proof. Proposition 1

Step 1) First we compare the minimum capital investment policy under three regimes. Compare the minimum capital investment policy under HC and LCM:

\[
\hat{q}_h = \frac{\hat{K}}{H - I + \hat{K}}, \quad \hat{q}_l = \frac{(1 - \gamma)\hat{K} + \gamma\beta(I - L) - \frac{\partial P(B)}{\partial q}C(-e^B) + P(B)C'(-e^B)\frac{\partial e^B}{\partial q}}{H - I + (1 - \gamma)\hat{K} - \gamma\alpha(H - I) + \gamma\beta(I - L)}
\]

Let \( a = \frac{(1 - \gamma)\hat{K} + \gamma\beta(I - L)}{H - I + (1 - \gamma)\hat{K} - \gamma\alpha(H - I) + \gamma\beta(I - L)} \) and \( b = \frac{\hat{K}}{H - I + \hat{K}} \). Then using assumption in (7), it can be shown that:

\[
a - b = \frac{\gamma(H - I)[\beta(I - L) - (1 - \alpha)\hat{K}]}{[H - I + (1 - \gamma)\hat{K} - \gamma\alpha(H - I) + \gamma\beta(I - L)] \cdot [H - I + \hat{K}]}
\]

\[
> \gamma(H - I)\beta(I - L)\frac{[1 - \frac{\hat{K}}{H + I - 2L}]}{[H - I + (1 - \gamma)\hat{K} - \gamma\alpha(H - I) + \gamma\beta(I - L)] \cdot [H - I + \hat{K}]}
\]

\[
= \frac{\gamma(H - I)\beta(I - L)\frac{H - L + D - L}{H + I - 2L}}{[H - I + (1 - \gamma)\hat{K} - \gamma\alpha(H - I) + \gamma\beta(I - L)] \cdot [H - I + \hat{K}]}
\]

\[
> 0 \quad \text{(risky debt is issued, } D > L) \quad (26)
\]

In addition, for any \( q \), the following conditions hold:

\[
\frac{\partial P(B)}{\partial q} = -[q(1 - \alpha) + (1 - q)\beta] < 0
\]

\[
\frac{\partial e^B}{\partial q} = \frac{2\beta(1 - \alpha)(H - L)}{[(1 + q)(1 - \alpha) + (1 - q)\beta]^2} > 0 \quad (27)
\]

Hence combining (26) and (27), we have \( \hat{q}_h < \hat{q}_l \).

Similarly we can also easily show that \( \hat{q}_f > \hat{q}_h \).

Now we need to compare \( \hat{q}_f \) with \( \hat{q}_l \). Compare the partial derivative of the objective function
with respect to $q$ at $\hat{K}$:

$$\frac{\partial}{\partial q} \Pi_l(q, \hat{K}) = \gamma[\beta(1-q)(I-L) - q(1-\alpha)(H-I)] + (1-\gamma)[\hat{K} - q(H - I + \hat{K})]$$

$$- \frac{\partial P(B)}{\partial q} C(u) + P(B)C'(u)\frac{\partial e}{\partial q}$$

$$\frac{\partial}{\partial q} \Pi_f(q, \hat{K}) = \gamma[(1-q)(I-L) - q(H - I)] + (1-\gamma)[\hat{K} - q(H - I + \hat{K})]$$

$$- \frac{\partial P(B)}{\partial q} C(u) + P(B)C'(u)\frac{\partial e}{\partial q}$$

The only difference in the partial derivative functions is the underlined part. Compare these two underlined parts:

$$[(1 - q)(I - L) - q(H - I)] - [\beta(1 - q)(I - L) - q(1 - \alpha)(H - I)]$$

$$= (I - L)(1 - q)(1 - \beta) - \alpha q(H - I)$$

$$< \frac{H - I}{H - I + \hat{K}}[(I - L)(1 - \beta) - \alpha \hat{K}] \text{ (since } \hat{q}_{(i,f)} > \frac{\hat{K}}{H - I + \hat{K}} \text{ holds)}$$

$$< 0 \text{ (by the assumption in (7))}$$

$$\Rightarrow \frac{\partial \Pi_f}{\partial q} < \frac{\partial \Pi_l}{\partial q}, \forall q$$

Therefore the optimal solution must satisfy $\hat{q}_f < \hat{q}_l$. This also extends to the result for any given level of capital requirement $k$.

Step 2) Now we need to show that the investment policy under LCM is always no more risky than the minimum capital investment policy, i.e, $q_l^* \geq \hat{q}_l$.

Define the function $\Gamma$ as:

$$\Gamma = \frac{\partial}{\partial q} \Pi_l(q, \hat{K}_l)$$

$$= \gamma[\beta(1 - \hat{q}_l)(I - L) - \hat{q}_l(1 - \alpha)(H - I)] + (1 - \gamma)[\hat{K}_l - \hat{q}_l(H - I + \hat{K}_l)]$$

$$- \frac{\partial P(B)}{\partial \hat{q}_l} C(u) + P(B)C'(u)\frac{\partial e}{\partial \hat{q}_l}$$

(28)

Since $\frac{\partial}{\partial \hat{K}} \Pi_l(\hat{q}_l, \hat{K}_l) = 0$ also holds at FOC, i.e,

$$-(1 - \gamma)\frac{(1 - \hat{q}_l)^2}{2} + P(B)(1 + k)C'(u(\hat{K}_l)) = 0$$

(29)

Substituting $P(B)C'(u)$ from (29) into the function of $\Gamma$ in (28), we have:
\[ \Gamma = \gamma \beta (1 - \hat{q}_l)(I - L) - \hat{q}_l (1 - \alpha)(H - I) + (1 - \gamma)[\hat{K}_l - \hat{q}_l(H - I + \hat{K}_l)] \]
\[ - \frac{\partial P(B)}{\partial \hat{q}_l} C(u) + \frac{(1 - \gamma)\hat{q}_l^2 \partial e^B}{2(1 + k) \partial \hat{q}_l} \]

Now taking the partial derivative of \( \Gamma \) with respect to \( \hat{K}_l \), we have:

\[ \frac{\partial \Gamma}{\partial \hat{K}_l} = (1 - \gamma)(1 - \hat{q}_l) + \frac{\partial P(B)}{\partial \hat{q}_l} C'(u) \] (31)

Then substituting the function of \( C'(u) \) from (29) into (31), and also substituting \( P(B) \) and \( \frac{\partial P(B)}{\partial \hat{q}_l} \) into (31), we have the following result:

\[ \frac{\partial \Gamma}{\partial \hat{K}_l} = (1 - \gamma)(1 - \hat{q}_l)[1 - \hat{q}_l^2 \frac{(1 - \alpha)(1 + \hat{q}_l)\beta}{(1 + k)((1 + \hat{q}_l)(1 - \alpha) + (1 - \hat{q}_l)\beta)}] > 0 \] (32)

Then taking the total derivative of the function \( \Gamma \) with respect to \( \hat{K}_l \), we have:

\[ \frac{\partial \Gamma}{\partial \hat{K}_l} + \frac{\partial \Gamma}{\partial \hat{q}_l} \frac{\partial \hat{q}_l}{\partial \hat{K}_l} = 0 \]

Given the second order condition in the proof of Lemma 2, we have \( \frac{\partial \Gamma}{\partial \hat{q}_l} < 0 \); and \( \frac{\partial \Gamma}{\partial \hat{K}_l} > 0 \) from (32), therefore we have:

\[ \frac{\partial \hat{q}_l}{\partial \hat{K}_l} > 0 \] (33)

Hence at the relaxed optimal solution, the higher equity capital always induces the less risky investment. Since \( \hat{K}_l \) and \( \hat{q}_l \) is also a set of solution that satisfies the FOC, it is easy to see that \( q^*_l \geq \hat{q}_l \) given \( K^*_l > K_l \).

Then combined with the result in Step 1), we have

\[ q^*_l > q^*_h. \]

Step 3) The next step is to compare \( q^*_l \) and \( q^*_f \). Following the proof in Step 1), we can also show that for any given equity capital \( K \), the optimal investment policy under fair value accounting is always more risky than under LCM, i.e. \( q^*_l(K) > q^*_f(K) \)

If under both regimes, the bank issues the minimum capital required, then \( q^*_f < q^*_l \) holds.

If under both regimes, the bank issues the capital in excess of the minimum requirement, we
need to compare $\hat{K}_l$ and $\hat{K}_f$ and the corresponding optimal investment policies.

Now since $\hat{q}_f$ and $\hat{K}_f$ satisfy the FOC condition for $K$, we have:

$$\frac{\partial}{\partial \hat{K}_f} \Pi_f(\hat{q}_f, \hat{K}_f) = -(1 - \gamma) \frac{(1 - \hat{q}_f)^2}{2} + (1 + k)P(B)C'(u) = 0$$

Under LCM, we also have the same form of FOC condition for $K$:

$$\frac{\partial}{\partial \hat{K}_l} \Pi_l(\hat{q}_l, \hat{K}_l) = -(1 - \gamma) \frac{(1 - \hat{q}_l)^2}{2} + (1 + k)P(B)C'(u) = 0$$

From the proof in 2), we have the following condition for the FOC solution under FV:

$$\frac{\partial^2}{\partial \hat{K}_f \partial \hat{q}_f} \Pi_f(\hat{q}_f, \hat{K}_f) > 0$$

In addition $q^*_l(\hat{K}_f) > \hat{q}_f(\hat{K}_f)$, therefore $\frac{\partial}{\partial \hat{K}_f} \Pi_f(q^*_l(\hat{K}_f), \hat{K}_f) > 0$

Given that FOC functions under FV and LCM have the same form:

$$\frac{\partial}{\partial \hat{K}_l} \Pi_l(q^*_l(\hat{K}_f), \hat{K}_f) > 0 \Rightarrow \hat{K}_l > \hat{K}_f, \hat{q}_l > \hat{q}_f$$

The only question remaining is about the likelihood of issuing equity capital in excess of the minimum requirement under two regimes. Suppose under fair accounting, the minimum capital $\hat{K}$ also satisfies the FOC, i.e,

$$\Lambda_f(\hat{K}, \hat{q}_f) = -(1 - \gamma) \frac{(1 - \hat{q}_f)^2}{2} + (1 + k)P(B)C'(-e^B(\hat{q}_f)) = 0$$

From the proof of Lemma 2, we know that at the optimal solution, the cross partial derivative $\frac{\partial^2 \Pi_f}{\partial q \partial K} > 0$, therefore we can get:

$$\Lambda_l(\hat{K}, \hat{q}_l) > 0, \text{ as } \hat{q}_l > \hat{q}_f$$

This means under LCM, the optimal solution for the bank is to issue equity in excess of the minimum requirement. Therefore, the bank is more likely to issue buffer capital under LCM than under FV.

Proof. Proposition 2

When $g' \to 0$, the bank will always exert effort of $a_j = 1$, such that the problem of the regulator
becomes:

$$\max_k W_j(k) = V(q_j^*(k)) + L(k)$$

s.t. \(q_j^*(k), K_j^*(k) \in \arg \max_{q,K} \Pi_j(q,K|k)\)

a. First show that under all regimes the optimal investment policy increases with \(k\), i.e, \(\partial q_j^*(k)/\partial k > 0\). This is obvious under HC. Under LCM and FV, we need to consider both the original and relaxed solutions. Although I only show the proof under LCM, the case for FV is similar.

(1) when \(q_l^*(k) = ˆq_l\), the FOC function at \( ˆK \) is given by:

$$\Gamma( ˆK) = (1 - \gamma)[ ˆK - ˆq(H - I + ˆK)] + \gamma[(1 - ˆq)\beta(I - L) - ˆq(1 - \alpha)(H - I)]$$

$$- \frac{\partial P(B)}{\partial q}C(-e^B) + P(B)C'(e^B)\frac{\partial e^B}{\partial q} = 0$$

$$\Rightarrow (1 - ˆq_l)\frac{\partial ˆK}{\partial k} + \frac{\partial ˆq}{\partial ˆq_l} = 0$$

$$\Rightarrow \frac{\partial ˆq}{\partial k} > 0 \quad \text{(Since } \frac{\partial ˆq}{\partial ˆq_l} < 0 \text{ from SOC) (34)}$$

(2) when \(q_l^*(k) = ˚q_l\), the optimal solution is interior solution, the FOC functions are defined as:

$$\Gamma = \frac{\partial}{\partial q} \Pi_l(˚q_l, ˚K_l), \quad \Lambda = \frac{\partial}{\partial K} \Pi_l(˚q_l, ˚K_l)$$

By Cramer’s rule, we can get:

$$\frac{\partial ˚q_l}{\partial k} = \left| \begin{array}{cc}
-\frac{\partial \Gamma}{\partial K} & -\frac{\partial \Gamma}{\partial ˚K_l} \\
\frac{\partial \Lambda}{\partial K} & -\frac{\partial \Lambda}{\partial ˚K_l} \\
\frac{\partial \Gamma}{\partial ˚q_l} & \frac{\partial \Gamma}{\partial ˚K_l} \\
\frac{\partial \Lambda}{\partial ˚q_l} & \frac{\partial \Lambda}{\partial ˚K_l}
\end{array} \right|$$

The denominator of the above equation is exactly the determinant of the Hessian Matrix, which by assumption is positive when \(C''\) is sufficiently large. Now we look at the numerator, we have the following conditions:
\[
\frac{\partial \Lambda}{\partial k} = P(B)(C'(u) + C''(u)D) > 0
\]
\[
\frac{\partial \Gamma}{\partial q} = -\frac{\partial P(B)}{\partial q}C'(u)D + P(B)C''(u)\frac{\partial e_B}{\partial q}D > 0
\]
\[
\frac{\partial \Gamma}{\partial K_l} > 0 \quad \text{(see the proof in Corollary 1)}
\]
\[
\frac{\partial \Lambda}{\partial K_l} < 0 \quad \text{(SOC for} \ K_l)\]

\[
\frac{\partial \Lambda}{\partial k} \frac{\partial \Gamma}{\partial K_l} - \frac{\partial \Gamma}{\partial k} \frac{\partial \Lambda}{\partial K_l} > 0
\]

Hence we can prove that:
\[
\frac{\partial q^*_j(k)}{\partial k} > 0 \quad \text{(35)}
\]

Therefore combining (34) and (35), we have \( \frac{\partial q^*_j(k)}{\partial k} > 0. \)

b. Next since the optimal \( k \) is determined by the FOC condition:
\[
W'_j(k) = \frac{\partial L(k)}{\partial k} + \frac{\partial V(q^*_j(k))}{\partial q} \frac{\partial q^*_j(k)}{\partial k} = 0
\]

As shown above \( \frac{\partial q^*_j(k)}{\partial k} > 0, \) and \( \frac{\partial L(k)}{\partial k} < 0, \) we must have \( \frac{\partial V(q^*_j(k))}{\partial q^*_j(k)} > 0 \) at the optimal \( k^*_j. \)

In addition,
\[
\frac{\partial^2 V(q)}{\partial q^2} - L - H < 0 \quad \text{(36)}
\]

Therefore at the optimal capital requirement, we must have \( q^*_j(k^*_j) < q_{fb}^l \)

b. Given that \( \forall k, q^*_j(k) > q^*_h(k) > q^*_f(k) \) and \( \frac{\partial V(q^*_j(k))}{\partial q^*_j(k)} > 0 \) (from \( q^*_j(k^*_j) < q_{fb}^l)), \) the following condition holds:

\[
V(q^*_j(k^*_j)) \geq V(q^*_h(k^*_h)) \Rightarrow W_f(k^*_h) \geq W_h(k^*_h)
\]
\[
V(q^*_j(k^*_j)) \geq V(q^*_j(k^*_f)) \Rightarrow W_f(k^*_f) \geq W_f(k^*_f)
\]

Therefore we must have \( W_f(k^*_j) \geq W_f(k^*_f) \geq W_f(k^*_h) \) \( \blacksquare \)

**Proof. Proposition 3**

When \( L'(k) \to 0, \) the problem in (10) becomes:

\[
\max_k a^*_j(k)V(q^*_j(k)) - g(a^*_j(k))
\]
\[
s.t. \quad a^*_j(k) \in \arg \max_a a \Pi_j(q^*_j(k), K^*_j(k), k) - g(a)
\]
\[
q^*_j(k), K^*_j(k) \in \arg \max_{q, K} \Pi_j(q, K|k)
\]

Substitute the bank’s solution to his own problem in (15) to get the optimal social welfare under
each accounting regime for any exogenous $k$:

$$W_j(k) = \frac{m}{2} \Pi_j(q_j^*(k), K_j^*(k), k)[V(q_j^*(k)) - \frac{1}{2} \Pi_j(q_j^*(k), K_j^*(k), k)]$$  \hspace{1cm} (37)$$

The maximum welfare under each accounting regime is given by $W_j(k_j^*)$, where $k_j^*$ is the optimal capital requirement for the regulator.

The maximum welfare under each accounting regime is given by $W_j(k_j^*)$, where $k_j^*$ is the optimal capital requirement for the regulator. The first step is to prove that $W_l(k_l^*) < W_h(k_h^*)$ and $W_l(k_l^*) < W_f(k_l^*)$ when $\gamma \to 1$. For any given $k$, we will show that $W_l(k) < W_h(k)$. Then it is easy to conclude that $W_l(k_l^*) < W_h(k_h^*)$.

Under HC, the welfare is:

$$W_h(k) = (1 - \gamma)\pi(q_h^*(k), K_h^*(k))[V(q_h^*(k)) - \frac{1}{2}\pi(q_h^*(k), K_h^*(k))]$$  \hspace{1cm} (38)$$

Under LCM, define $\Delta_l = E[e_l] - E[C(u_l)]$, then the welfare function becomes:

$$W_l(k) = (1 - \gamma)\pi(q_l^*(k), K_l^*(k))[V(q_l^*(k)) - \frac{1}{2}\pi(q_l^*(k), K_l^*(k))]$$

$$-\Delta_l[(1 - \gamma)\pi(q_l^*(k), K_l^*(k)) - V(q_l^*(k))]$$  \hspace{1cm} (39)$$

Define $U(q, K) = \pi(q, K)[V(q) - \frac{1}{2}\pi(q, K)]$, and take derivative with respect to $q$ and $K$. It can be shown that for any $q_h^* < q < q_f^*$, the following holds when $\gamma \to 1$:

$$\frac{\partial U(q, K)}{\partial K} \leq 0, \quad \frac{\partial U(q, K)}{\partial q} \leq 0$$

Therefore since $q_h^*(k) > q_h^*(k)$ and $K_h^*(k) \geq K_h^*(k)$, we have

$$U(q_l^*, K_l^*) < U(q_h^*, K_h^*)$$  \hspace{1cm} (40)$$

In addition, under lower of cost or market accounting, we have $\Delta_l < 0$. And for any $q$, we can show that $\pi(q, K) < V(q)$. Therefore in (39), we have $\Delta_l[(1 - \gamma)\pi(q_l^*(k), K_l^*(k)) - V(q_l^*(k))] > 0$. Combined with the result in (40), we have for any given $k$,

$$W_l(k) < W_h(k)$$  \hspace{1cm} (41)$$

To compare the welfare under FV and HC, define $\Delta_f = E[e_f] - E[C(u_f)]$, then the result depends on the sign of $\Delta_f$. When the cost and marginal cost of violating regulatory constraint are
sufficiently high, $\Delta f < 0$; otherwise, $\Delta f > 0$. In the former case, comparing FV to HC is similar to the proof shown above for LCM, i.e., $W_f(k_f^*) < W_h(k_h^*)$. In the latter case, the result will be opposite; we can show that $W_f(k) > W_h(k)$ following the same process. Therefore under these conditions, we have $W_h(k_h^*) < W_f(k_f^*).$

**Proof. Proposition 4** The proof of Proposition 4 follows similar steps as Proposition 1. From Lemma 5, we know that the bank may issue buffer capital above the minimum level under the mixed attribute accounting, while the bank only issues the minimum capital under historical cost accounting, i.e., $K_m^* \geq K_h^* = ˚K$.

First we compare the minimum capital investment policy ($q_m^*$) under the mixed attribute accounting regime with two pure accounting regimes. From first order conditions with respect to $K$, we get:

$$\hat{q}_h = \frac{K}{H - I + K}$$

$$\hat{q}_f = \frac{(1 - \gamma)K + \gamma(I - L) - \frac{\partial P(B)}{\partial q} C(-e^B) + P(B)C'(e^B)\frac{\partial e^B}{\partial q}}{H - I + (1 - \gamma)K + \gamma(I - L)}$$

$$\hat{q}_m = \frac{(1 - \gamma)K + \gamma \beta(I - L) - (1 - m)\frac{\partial P(B)}{\partial q} C(-e^B) + (1 - m)P(B)C'(e^B)\frac{\partial e^B}{\partial q}}{H - I + (1 - \gamma)K - \gamma \alpha(H - I) + \gamma \beta(I - L)}$$

We have shown that for any $q$, the following conditions hold:

$$\frac{\partial P(B)}{\partial q} < 0, \quad \frac{\partial e^B}{\partial q} > 0$$

Following the proof in Proposition 1, we have $\hat{q}_h < \hat{q}_m < \hat{q}_f$.

Similar to Step 2) in Proposition 1, we can show that the unconstrained optimal investment policy is increasing in the unconstrained optimal capital, i.e., $\frac{\partial q}{\partial K} > 0$. From Lemma 5, when the bank issues buffer capital above the minimum capital level ($K_m > ˚K$), $\hat{q}_m > \hat{q}_m > \hat{q}_h$. Therefore the bank’s optimal investment policy under the mixed attribute accounting is always less risky than under historical cost accounting.

Next we compare the unconstrained optimization solutions under the mixed attribute accounting ($q_m(K), K_m$) with the unconstrained optimal solutions under the fair value accounting ($q_f(K), K_f$). For any $K$, we know that $\hat{q}_m(K) < \hat{q}_f(K)$ always holds from FOC with respect to $q$.

From the FOC condition with respect to $K$, we get:

$$\frac{\partial}{\partial K} \Pi_m(q, K) = -(1 - \gamma)(1 - q)^2 + (1 - m)(1 + k)P(B)C'(u) = 0$$

$$\frac{\partial}{\partial K} \Pi_f(q, K) = -(1 - \gamma)(1 - q)^2 + P(B)(1 + k)C'(u) = 0$$

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Therefore $\frac{\partial}{\partial K} \Pi_m(q, K) < \frac{\partial}{\partial K} \Pi_f(q, K)$. For any $q$, $\dot{K}_m(q) < \dot{K}_f(q)$. Combining the two first order conditions, we have $\dot{q}_m < \dot{q}_f$ and $\dot{K}_m < \dot{K}_f$.

Hence $q^*_f > q^*_m > q^*_h$ and $K^*_f \geq K^*_m \geq K^*_h$. ■

**Proof. Proposition 6**

Since the endogenous cost function is now given by $E[C(u)] = E[\phi^* \delta(u) A_0]$, we only need to prove that $C'(u) > 0$ and $C''(u) > 0$ holds. Then following the proof of Proposition 1, we can infer that Proposition 6 holds.

Substituting $\delta^* = \frac{u}{A_0(k - \delta(u)(1 + k))}$ into the cost function above, we have

$$C(u) = \frac{u \delta(u)}{k - \delta(u)(1 + k)}$$

Taking the first order derivative with respect to $u$, and using the assumptions $\delta'(u) > 0$ and $\delta''(u) > 0$, we obtain that

$$\frac{\partial C(u)}{\partial u} = \frac{\delta'(u) + u}{k - \delta(u)(1 + k)} + \frac{u \delta(u)(1 + k) \delta'(u)}{(k - \delta(u)(1 + k))^2} > 0$$

and furthermore,

$$\frac{\partial^2 C(u)}{\partial u^2} > 0$$

Therefore following the proof in Proposition 1, we can prove that under fair value accounting regime, the optimal investment policy $q^*_f > q^*_h$, and $K^*_f \geq K^*_h$. ■